Rotational Motion

Question1

The angular momentum of a rotating body is ' L '. When the frequency of rotating body is tripled and its kinetic energy is made one-third, the new angular momentum becomes

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Options:

A.

$\frac{1}{9}$ L

B.

$\frac{1}{3}$ L

C.

6 L

D.

9 L

Answer: A

Solution:

Given:

- Angular momentum of a rotating body initially = L.
- Frequency of rotation is tripled $(f \rightarrow 3f)$.
- Kinetic energy becomes one-third of its initial value.

We want: new angular momentum in terms of L.

Step 1: General relations

- Angular velocity: $\omega = 2\pi f$.
- Rotational KE: $K = \frac{1}{2}I\omega^2$.
- Angular momentum: $L = I\omega$.

Step 2: Initial conditions

- Initial angular momentum: $L = I\omega$.
- Initial KE: $K = \frac{1}{2}I\omega^2$.

Step 3: After changes

New angular velocity: $\omega' = 3\omega$.

Now required KE is one third the original:

$K' = \frac{1}{3}K = \frac{1}{3} \cdot \frac{1}{2}I\omega^2$.

But also,

$K' = \frac{1}{2}I'(\omega')^2 = \frac{1}{2}I'(3\omega)^2 = \frac{9}{2}I'\omega^2$.

Equating,

$\frac{9}{2}I'\omega^2 = \frac{1}{3} \cdot \frac{1}{2}I\omega^2$.

Cancel $\frac{1}{2}\omega^2$:

$9I' = \frac{1}{3}I$,

$I' = \frac{I}{27}$.

So, the moment of inertia must reduce.

Step 4: New angular momentum

$L' = I'\omega' = \left(\frac{I}{27}\right)(3\omega) = \frac{I\omega}{9}$.

But $I\omega = L$.

So,

$L' = \frac{L}{9}$.

✅ Final Answer

$\boxed{\frac{1}{9}L}$

Correct Option: A

Question2

The moment of inertia of a thin uniform rod of mass ' M ' and length ' L ', about an axis perpendicular to length of the rod and at a distance ' $L/4$ ' from one end is

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Options:

A.

$\frac{ML^2}{6}$

B.

$\frac{ML^2}{12}$

C.

$\frac{7ML^2}{24}$

D.

$\frac{7ML^2}{48}$

Answer: D

Solution:

The moment of inertia of the rod about an axis passing through the centre and perpendicular to its length is given by,

$I_0 = \frac{ML^2}{12}$

A point at a distance $\frac{L}{4}$ from its end will also be at a distance $\frac{L}{4}$ from the centre.

Hence by parallel axis theorem,

$I = I_0 + M\left(\frac{L}{4}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$

Question3

A same torque is applied to a disc and a ring of equal mass and radii then

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Options:

A. the ring will rotate with greater angular frequency.

B. both will rotate with same angular frequency.

C. the disc will rotate with greater angular frequency.

D. both will rotate with same angular velocity.

Answer: C

Solution:

Moment of inertia of disc $\mathrm{t_d} = \frac{\mathrm{MR}^2}{2}$

Moment of inertia of ring $\mathrm{I_r} = \mathrm{MR}^2$

$\therefore \quad I_r > I_d$

Angular acceleration $\alpha = \frac{\tau}{1}$

As torque is same, $\alpha \propto \frac{1}{1}$

$\because \quad \alpha_d > \alpha_r$

$\therefore$ Disc will rotate with greater angular frequency

Question4

A solid sphere at rests rolls down an inclined plane of vertical height h without sliding. Its speed on reaching the bottom of plane is ($\mathrm{g} =$ acceleration due to gravity)

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Options:

A.

$\left(\frac{9\text{gh}}{11}\right)^{\frac{1}{2}}$

B.

$\left(\frac{10gh}{7}\right)^{\frac{1}{2}}$

C.

$\left(\frac{8\text{gh}}{7}\right)^{\frac{1}{2}}$

D.

$\left(\frac{6\text{gh}}{7}\right)^{\frac{1}{2}}$

Answer: B

Solution:

Step 1: Energy conservation

At the top, the sphere has potential energy:

$$E_{\text{top}} = mgh$$

At the bottom, the energy is entirely kinetic (translational + rotational):

$$E_{\text{bottom}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Step 2: Moment of inertia of a solid sphere

For a solid sphere about its diameter:

$$I = \frac{2}{5}mR^2$$

Also, rolling without slipping condition:

$$v = \omega R \quad \Rightarrow \quad \omega = \frac{v}{R}$$

Step 3: Substitute

$$E_{\text{bottom}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

Step 4: Equating to initial energy

$mgh = \frac{7}{10}mv^2$

Cancel m:

$gh = \frac{7}{10}v^2$

$v^2 = \frac{10}{7}gh$

$v = \sqrt{\frac{10gh}{7}}$

✅ **Final Answer:**

The speed of the sphere at the bottom is:

$$\boxed{\left(\frac{10gh}{7}\right)^{1/2}}$$

So the correct option is **B**.

Question5

A man standing on a turn-table is rotating at a certain angular frequency with his arms outstretched. He suddenly folds his arms. If his moment of inertia with folded arms is 75% of that with outstretched arms, then his rotational kinetic energy will

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Options:

A.

increase by 33.3%

B.

decrease by 33.3%

C.

increase by 25.0%

D.

decrease by 25.0%

Answer: A

Solution:

Conservation of Angular Momentum

When the man folds his arms, his total spin (angular momentum) does not change because no outside force acts on him.

Mathematically, $I_1\omega_1 = I_2\omega_2$ We know: $I_2 = 75\% \text{ of } I_1 = \frac{3}{4}I_1$

So, $I_1\omega_1 = \frac{3}{4}I_1\omega_2$

Divide both sides by I_1: $\omega_1 = \frac{3}{4}\omega_2$

So, $\omega_2 = \frac{4}{3}\omega_1$

Rotational Kinetic Energy

When his arms are outstretched: $K.E_1 = \frac{1}{2}I_1\omega_1^2$

When his arms are folded: $K.E_2 = \frac{1}{2}I_2\omega_2^2$

Substitute $I_2 = \frac{3}{4}I_1$ and $\omega_2 = \frac{4}{3}\omega_1$: $K.E_2 = \frac{1}{2}(\frac{3}{4}I_1)(\frac{4}{3}\omega_1)^2$

Now, $(\frac{4}{3})^2 = \frac{16}{9}$, so: $K.E_2 = \frac{1}{2} \times \frac{3}{4}I_1 \times \frac{16}{9}\omega_1^2$

Simplify: $K.E_2 = \frac{1}{2}I_1\omega_1^2 \times \frac{3}{4} \times \frac{16}{9}$

Multiply the fractions: $\frac{3}{4} \times \frac{16}{9} = \frac{48}{36} = \frac{4}{3}$

So, $K.E_2 = \frac{4}{3}K.E_1$

Percentage Increase in Kinetic Energy

Change in energy: $\text{Increase} = K.E_2 - K.E_1 = \frac{4}{3}K.E_1 - K.E_1 = \frac{1}{3}K.E_1$

Percentage increase: $\text{Percentage} = \frac{1}{3} \times 100 = 33.3\%$

Final Answer

When the man folds his arms, his rotational kinetic energy increases by 33.3%.

Question6

A rigid body rotates about a fixed axis with variable angular velocity $(\alpha - \beta t)$ at time t, where α and β are constants. The angle through which it rotates before it comes to rest is

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Options:

A. $\frac{\alpha}{\beta}$

B. $\frac{\alpha^2}{\beta}$

C. $\frac{\alpha^2}{2\beta}$

D. $\frac{\alpha}{2\beta}$

Answer: C

Solution:

Angular velocity of rigid body

$\omega(t) = \alpha - \beta t$

when body is at rest

$$\begin{aligned} \omega &= \alpha - \beta t = 0 \\ \Rightarrow t &= \frac{\alpha}{\beta} \end{aligned}$$

Angle rotated is the integral of angualr velocity.

$$\theta = \int_0^{\alpha\beta} \alpha(t) dt = \int_0^{\alpha\beta} (\alpha - \beta t) dt$$

$$\theta = \left[\alpha t - \frac{\beta t^2}{2}\right]_0^{\alpha/\beta}$$

$$\theta = \frac{\alpha^2}{\beta} - \frac{\alpha^2}{2\beta}$$

$$\theta = \frac{\alpha^2}{2\beta}$$

Question7

Moment of inertia of a solid sphere about its diameter is 'I'. It is then casted into 27 small spheres of same diameter. The moment of inertia of each small sphere about its diameter is

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Options:

A. $\frac{1}{44}$

B. $\frac{\mathrm{I}}{188}$

C. $\frac{1}{204}$

D. $\frac{1}{243}$

Answer: D

Solution:

$\mathrm{I} = \frac{2}{5}\mathrm{MR}^2$

For smaller spheres mass $\mathrm{M}' = \frac{\mathrm{M}}{27}$

Radius of each smaller sphere $= \mathrm{R}' = \frac{\mathrm{R}}{3}$

$$\begin{aligned} \therefore \quad I' &= \frac{2}{5}M'R'^2 = \frac{2}{5} \times \frac{M}{27} \times \frac{R^2}{9} \\ &= \frac{1}{243}\left(\frac{2}{5}M^2\right) = \frac{1}{243} \end{aligned}$$

Question8

Moment of inertia of the rod about an axis passing through the centre and perpendicular to its length is ' I_1 '. The same rod is bent into a ring and its moment of inertia about the diameter is ' I_2 '. Then I_1/I_2 is

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Options:

A. $\frac{3\pi^2}{2}$

B. $\frac{2\pi^2}{3}$

C. $\frac{\pi^2}{3}$

D. $\frac{\pi^2}{9}$

Answer: B

Solution:

Step 1: Moment of inertia of the straight rod

The moment of inertia for a rod of mass M and length L, about its center and perpendicular to its length, is given by:

$I_1 = \frac{ML^2}{12}$

Step 2: Bending the rod into a ring

When you bend the rod into a circle (a ring), the length of the rod becomes the circumference of the ring.

$L = 2\pi r$

This means the radius r of the ring is:

$r = \frac{L}{2\pi}$

Step 3: Moment of inertia of the ring about its diameter

The moment of inertia of a ring of mass M and radius r about its diameter is:

$I_2 = \frac{Mr^2}{2}$

Now, substitute the value of r from above:

$I_2 = \frac{M}{2}\left(\frac{L}{2\pi}\right)^2 = \frac{M}{2} \cdot \frac{L^2}{4\pi^2} = \frac{ML^2}{8\pi^2}$

Step 4: Ratio of the two moments of inertia

To find $\frac{I_1}{I_2}$, divide the two results:

$\frac{I_1}{I_2} = \frac{\frac{ML^2}{12}}{\frac{ML^2}{8\pi^2}} = \frac{ML^2}{12} \times \frac{8\pi^2}{ML^2} = \frac{8\pi^2}{12} = \frac{2\pi^2}{3}$

Question9

A bob of mass ' m ' is tied by a massless string whose other end is wound on a flywheel (disc) of radius ' R ' and mass ' m '. When released from the rest, the bob starts falling vertically downwards. If the bob has covered a vertical distance ' h ', then angular speed of wheel will be (There is no slipping between string and wheel, g - acceleration due to gravity)

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Options:

A. $\frac{2}{\mathrm{R}}\sqrt{\frac{\mathrm{gh}}{3}}$

B. $\frac{1}{\mathrm{R}}\sqrt{\frac{2\mathrm{gh}}{3}}$

C. $\mathrm{R}\sqrt{\frac{2\mathrm{gh}}{3}}$

D. $2R\sqrt{\frac{gh}{3}}$

Answer: A

Solution:

According to law of conservation of energy,

$$\mathrm{P.E} = \mathrm{K.E}$$

$$\therefore \quad \mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{I}\omega^2$$

$$\therefore \quad \mathrm{mgh} = \frac{1}{2}\,\mathrm{m}(\omega r)^2 + \frac{1}{2}\left(\frac{\mathrm{mr}^2}{2}\right)\omega^2$$

$$\ldots\left(\because \mathrm{v} = \mathrm{r}\omega, \mathrm{I} = \frac{\mathrm{mr}^2}{2}\right)$$

$$\therefore \quad \mathrm{mgh} = \tfrac{3}{4}\,\mathrm{m}^2\mathrm{r}^2$$

$$\therefore \quad \mathrm{gh} = \tfrac{3}{4}\omega^2\mathrm{r}^2$$

$$\therefore \quad \omega = \sqrt{\tfrac{4\mathrm{gh}}{3\mathrm{r}^2}} = \tfrac{2}{\mathrm{r}}\sqrt{\tfrac{\mathrm{gh}}{3}}$$

Question10

A thin uniform rod of length ' L ' and mass ' M ' is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ' ω '. Its centre of mass rises to a maximum height of

(g = acceleration due gravity)

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Options:

A. $\frac{L^2\omega^2}{2g}$

B. $\frac{\mathrm{L}\omega}{6\ \mathrm{g}}$

C. $\frac{\mathrm{L}\omega}{2\ \mathrm{g}}$

D. $\frac{\mathrm{L}^2\omega^2}{6\ \mathrm{g}}$

Answer: D

Solution:

Step 1: Concept

- The rod swings like a physical pendulum about one end.
- When its angular speed is **maximum**, it must be passing through the lowest vertical position (since at the lowest position, kinetic energy is max and potential energy min).
- At the highest point, angular speed becomes zero (purely potential energy).

Thus, energy conservation applies:

Maximum rotational KE = Gain in gravitational PE of CM.

Step 2: Energy expressions

- For a rod of mass M, length L, pivot at one end:

Moment of inertia about one end is

$I = \frac{1}{3} M L^2$.

- Rotational KE when angular speed $= \omega$:

$K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{3} M L^2 \omega^2 = \frac{1}{6} M L^2 \omega^2$.

- Rise in potential energy of centre of mass:
- CM is initially below pivot by distance $L/2$.
- If it rises a vertical height h, gravitational potential energy increases by:

$\Delta U = M g h$.

Step 3: Apply conservation of energy

$\frac{1}{6} M L^2 \omega^2 = M g h$.

$h = \frac{L^2 \omega^2}{6 g}$.

Step 4: Final Answer

$\boxed{\frac{L^2 \omega^2}{6 g}}$

That corresponds to **Option D**. ✅

Question11

Using Bohr's quantisation condition, what is the rotational energy in the second orbit for a diatomic molecule?

($I =$ moment of inertia of diatomic molecule and $\mathrm{h} =$ Planck's constant)

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Options:

A. $\frac{\mathrm{h}}{2 \mathrm{I} \pi^2}$

B. $\frac{h^2}{2 I \pi^2}$

C. $\frac{\mathrm{h}^2}{2 \mathrm{I}^2 \pi^2}$

D. $\frac{\mathrm{h}}{2\mathrm{I}^2\pi}$

Answer: B

Solution:

Step 1: Recall the quantisation condition

Bohr's quantisation condition for angular momentum:

$$L = n\frac{h}{2\pi}, \quad n = 1, 2, 3, \ldots$$

Step 2: Express the rotational energy

Rotational kinetic energy is:

$$E = \frac{L^2}{2I}$$

Substitute $L = n\frac{h}{2\pi}$:

$$E_n = \frac{1}{2I}\left(n\frac{h}{2\pi}\right)^2$$

$$E_n = \frac{n^2h^2}{8\pi^2 I}$$

Step 3: For the second orbit (n=2)

$$E_2 = \frac{(2)^2h^2}{8\pi^2 I} = \frac{4h^2}{8\pi^2 I} = \frac{h^2}{2I\pi^2}$$

✅ **Final Answer:**

$$\boxed{\text{Option B: } \frac{h^2}{2I\pi^2}}$$

Question12

The moment of inertia of a solid sphere of mass ' m ' and radius ' R ' about its diametric axis is ' I '. Its moment of inertia about a tangent in the plane is

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Options:

A. 2.5 I

B. 3.0 I

C. 3.5 I

D. 4 I

Answer: C

Solution:

M.I. of sphere about the diameter $= \frac{2}{5}\mathrm{MR}^2$

$\frac{2}{5}\mathrm{MR}^2 = \mathrm{I}$ or $\mathrm{MR}^2 = \frac{5}{2}\mathrm{I}$

According to theorem of parallel axes, M.I. about the tangent

$= I + \frac{5}{2}I = \frac{7}{2}I = 3.5I$

Question13

Two discs of moment of inertia ' I_1 ' and ' I_2 ' and angular speeds ' ω_1 ' and ' ω_2 ' are rotating along the collinear axes passing through their centre of mass and perpendicular to their plane. If the two discs are made to rotate together along the same axis. The rotational kinetic energy of the system will be

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Options:

A. $\frac{I_1\omega_1+I_2\omega_2}{2(I_1+I_2)^2}$

B. $\frac{(I_1\omega_1-I_2\omega_2)^2}{2(I_1+I_2)}$

C. $\frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1-I_2)}$

D. $\frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$

Answer: D

Solution:

Step 1: Conservation principle

When the two discs are suddenly coupled (say by clamping them together along the same axis),

- **Angular momentum is conserved** (because external torque about the axis is zero).
- **Kinetic energy is not necessarily conserved** (some energy may be lost as heat/friction during coupling).

Step 2: Total angular momentum before coupling

For disc 1:

$L_1 = I_1\omega_1$

For disc 2:

$L_2 = I_2\omega_2$

So, total angular momentum:

$L = I_1\omega_1 + I_2\omega_2$

Step 3: Common angular velocity after coupling

When they rotate together: Let the final angular velocity = ω.

Moment of inertia = $I_1 + I_2$.

From conservation of angular momentum:

$(I_1 + I_2)\,\omega = I_1\omega_1 + I_2\omega_2$

$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

Step 4: Final rotational kinetic energy

$K = \frac{1}{2}(I_1 + I_2)\,\omega^2$

Substitute ω from above:

$K = \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$

$K = \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$

Final Answer:

$$\boxed{\frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}}$$

That corresponds to **Option D**.

Question14

Four particles each of mass M are placed at the corners of a square of side L . The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is

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Options:

A. $\frac{L}{2}$

B. $\frac{L}{\sqrt{2}}$

C. 2 L

D. $\frac{L}{4}$

Answer: B

Solution:

Step 1: Recall definition of radius of gyration

Radius of gyration is defined by:

$$k = \sqrt{\frac{I}{M_{\text{total}}}}$$

where I = moment of inertia of the system about the given axis, and M_{total} is the total mass.

Step 2: Distance of each mass from axis

The axis is perpendicular to the square and passes through its centre.

So, the distance of each mass (corner of square) from centre = half of the diagonal =

$$r = \frac{\sqrt{2}}{2} L = \frac{L}{\sqrt{2}}$$

Step 3: Moment of inertia

Each mass contributes Mr^2 where $r = L/\sqrt{2}$.

So,

$$I = 4 \cdot M\left(\frac{L}{\sqrt{2}}\right)^2 = 4M \cdot \frac{L^2}{2} = 2ML^2$$

Step 4: Total mass

$$M_{\text{total}} = 4M$$

Step 5: Radius of gyration

$$k = \sqrt{\frac{I}{M_{\text{total}}}} = \sqrt{\frac{2ML^2}{4M}} = \sqrt{\frac{L^2}{2}} = \frac{L}{\sqrt{2}}$$

✅ **Final Answer:**

$$\boxed{\frac{L}{\sqrt{2}}}$$

That corresponds to **Option B**.

Question15

What is the linear velocity if angular velocity $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and radius $\vec{r} = (5\hat{i} - 6\hat{j} + 6\hat{k})$?

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Options:

A. $(-30\hat{i} - 13\hat{j} - 38\hat{k})$

B. $(8\hat{i} - 10\hat{j} + 7\hat{k})$

C. $(-18\hat{i} - 13\hat{j} + 2\hat{k})$

D. $(-2\hat{i} - 2\hat{j} - 5\hat{k})$

Answer: C

Solution:

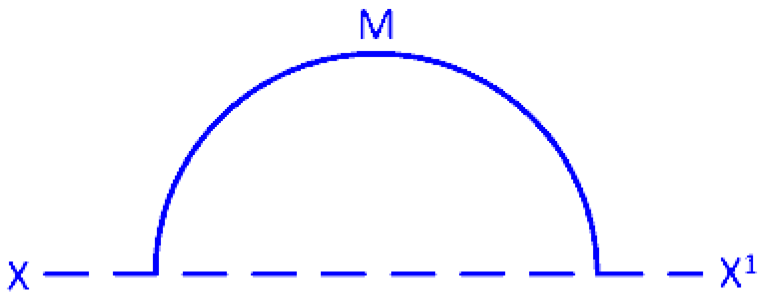
$$\vec{v}=\vec{\omega}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -4 & 1\\ 5 & -6 & 6\end{vmatrix}$$

$$=\hat{i}(-24+6)-\hat{j}(18-5)+\hat{k}(-18+20)$$

$$\vec{v}=-18\hat{i}-13\hat{j}+2\hat{k}$$

Question16

A thin metal wire of length ' L ' and mass ' M ' is bent to form semicircular ring as shown. The moment of inertia about XX' is



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Options:

A. $\frac{\mathrm{ML}^2}{4\pi^2}$

B. $\frac{2ML^2}{\pi^2}$

C. $\frac{\mathrm{ML}^2}{2\pi^2}$

D. $\frac{\mathrm{ML}^2}{\pi^2}$

Answer: C

Solution:

Both the semi-circular arc of a ring and the complete ring have the same moment of inertia because the mass in both cases is distributed at the same radial distance R from the central axis. Thus, the moment of inertia is determined solely by the distance R and the total mass M , not by the shape of the distribution along the ring.

M.I. of ring about diameter $\mathrm{I}=\frac{\mathrm{MR}^2}{2}\ldots..$ (i)

$\because \quad \mathrm{L}=\pi\mathrm{R}\Rightarrow\mathrm{R}=\mathrm{L}/\pi$

$\therefore$ From equation (i), $I = \frac{ML^2}{2\pi^2}$

Question17

A solid cylinder of mass ' M ' and radius ' R ' is rotating about its geometrical axis. A solid sphere of same mass and same radius is also rotating about its diameter with an angular speed half that of the cylinder. The ratio of the kinetic energy of rotation of the sphere to that of the cylinder will be

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Options:

A. 2 : 3

B. 3 : 2

C. 1 : 5

D. 5 : 1

Answer: C

Solution:

Step 1: Recall the moments of inertia

- Solid cylinder about its symmetry axis:

$I_{\text{cyl}} = \frac{1}{2}MR^2$

- Solid sphere about its diameter:

$I_{\text{sphere}} = \frac{2}{5}MR^2$

Step 2: Rotational KE formula

Rotational KE $= \frac{1}{2}I\omega^2$

Step 3: Compute KEs

- Cylinder (angular speed = ω):

$K_{\text{cyl}} = \frac{1}{2} \cdot \frac{1}{2} MR^2 \cdot \omega^2 = \frac{1}{4} MR^2 \omega^2$

- Sphere (angular speed $= \frac{\omega}{2}$):

$K_{\text{sphere}} = \frac{1}{2} \cdot \frac{2}{5} MR^2 \cdot \left(\frac{\omega}{2}\right)^2$

$= \frac{1}{2} \cdot \frac{2}{5} MR^2 \cdot \frac{\omega^2}{4} = \frac{1}{20} MR^2 \omega^2$

Step 4: Ratio

$$\frac{K_{\text{sphere}}}{K_{\text{cyl}}} = \frac{\frac{1}{20} MR^2 \omega^2}{\frac{1}{4} MR^2 \omega^2} = \frac{1}{20} \cdot \frac{4}{1} = \frac{1}{5}$$

So the ratio = 1 : 5.

✅ **Final Answer:**

Option C: 1 : 5

Question18

A solid sphere of mass ' m ' and radius ' R ' is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with angular speed twice that of sphere. The ratio of kinetic energy of sphere to kinetic energy of cylinder will be

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Options:

A. 2 : 3

B. 1 : 5

C. 3 : 1

D. 1 : 4

Answer: B

Solution:

$I_{\text{sphere}} \quad = I_s = \frac{2}{5} mR^2$

Let ω_s be angular speed of sphere,

$$\begin{aligned} \therefore \quad E_{\text{sphere}} & =\frac{1}{2} I_s \omega_s^2 \\ & =\frac{1}{2}\left(\frac{2}{5} m R^2\right) \omega_s^2 \quad \quad \ldots(i) \end{aligned}$$

Similarly,

$\mathrm{I}_{\text{cylinder}} \quad = \mathrm{I}_c = \frac{1}{2} \mathrm{mR}^2$

Let ω_c be the angular speed of cylinder, Then it is given

$$\begin{aligned} & \omega_c=2 \omega_s \\ \therefore \quad \mathrm{E}_{\text{cylinder}} & =\frac{1}{2} \mathrm{I}_c \omega_c^2 \\ & =\frac{1}{2}\left(\frac{1}{2} \mathrm{mR}^2\right)\left(2 \omega_s\right)^2 \quad \ldots \text{(ii)} \\ \therefore \quad \frac{\mathrm{E}_{\text{sphere}}}{\mathrm{E}_{\text{cylinder}}} & =\frac{\frac{1}{2}\left(\frac{2}{5} \mathrm{mR}^2\right) \omega_s^2}{\frac{1}{2}\left(\frac{1}{2} \mathrm{mR}^2\right)\left(4 \omega_s^2\right)} \quad \ldots[\text{ From (i) and (ii) }] \\ & =\frac{1}{5} \end{aligned}$$

Question19

A solid sphere and thin walled hollow sphere have same mass and same material. Which of them have greater moment of inertia about their diameter?

[I_{h} = moment of inertia of hollow sphere about an axis coinciding with its diameter, I_5 = moment of inertia of solid sphere about an axis coinciding with its diameter]

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Options:

A. $\mathrm{I}_{\mathrm{s}} > \mathrm{I}_{\mathrm{h}}$

B. $I_h \geqslant I_s$

C. $\mathrm{I}_{\mathrm{h}} > \mathrm{I}_{\mathrm{s}}$

D. $I_h = I_s$

Answer: C

Solution:

$$\therefore \quad \mathrm{I_s} = \frac{2}{5}\mathrm{MR}^2, \mathrm{I_h} = \frac{2}{3}\mathrm{MR}^2$$
$$\therefore \quad \frac{\mathrm{I_s}}{\mathrm{I_h}} = \frac{\left(\frac{2}{5}\mathrm{MR}^2\right)}{\left(\frac{2}{3}\mathrm{MR}^2\right)} = \frac{3}{5} < 1$$
$$\therefore \quad \frac{\mathrm{I_s}}{\mathrm{I_h}} < 1 \Rightarrow \mathrm{I_h} > \mathrm{I_s}$$

Question20

If force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acts along $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$ then the torque acting at that point is

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Options:

A. $(14\hat{\mathrm{i}} - 38\hat{\mathrm{j}} + 16\hat{\mathrm{k}})$

B. $(-14\hat{i} + 34\hat{j} - 16\hat{k})$

C. $(21\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 4\hat{\mathrm{k}})$

D. $(4\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 6\hat{\mathrm{k}})$

Answer: A

Solution:

$$\begin{aligned} \vec{\tau} = \vec{r} \times \vec{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} \\ &= \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9) \\ &= (14\hat{i} - 38\hat{j} + 16\hat{k})\mathrm{N-m} \end{aligned}$$

Question21

The moment of inertia of a ring about an axis passing through its centre and perpendicular to its plane is I. It is rotating with angular velocity ω. Another identical ring is gently placed on it so that their centres coincide. If both the rings are rotating about the same axis then loss in kinetic energy is

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Options:

A. $\frac{I\omega^2}{16}$

B. $\frac{I\omega^2}{8}$

C. $\frac{I\omega^2}{4}$

D. $\frac{I\omega^2}{2}$

Answer: C

Solution:

Step 1: Initial kinetic energy

Only first ring is rotating:

$KE_{\text{initial}} = \frac{1}{2} I\omega^2$

Step 2: Conservation of angular momentum

System is isolated (no external torque about axis).

Let final angular velocity = ω_f.

Initial angular momentum:

$L_{\text{initial}} = I\omega$

Final total moment of inertia = $I + I = 2I$.

So,

$L_{\text{final}} = (2I)\,\omega_f$

By conservation:

$I\omega = 2I\,\omega_f \quad \Rightarrow \quad \omega_f = \frac{\omega}{2}$

Step 3: Final kinetic energy

$KE_{\text{final}} = \frac{1}{2}(2I)\left(\frac{\omega}{2}\right)^2 = I \cdot \frac{\omega^2}{4} = \frac{I\omega^2}{4}$

Step 4: Loss in kinetic energy

$\Delta KE = KE_{\text{initial}} - KE_{\text{final}} = \frac{1}{2}I\omega^2 - \frac{1}{4}I\omega^2 = \frac{1}{4}I\omega^2$

✅ **Final Answer:**

$\boxed{\frac{I\omega^2}{4}}$

That corresponds to **Option C**.

Question22

A body is rotating about its own axis. Its rotational kinetic energy is ' x ' and its angular momentum is ' y '. Hence its moment of inertia about its own axis is

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Options:

A. $\frac{x^2}{2y}$

B. $\frac{y^2}{2x}$

C. $\frac{x}{2y}$

D. $\frac{y}{2x}$

Answer: B

Solution:

We are asked about rotational kinetic energy x, angular momentum y, and the moment of inertia I.

Step 1: Recall formulas

- Rotational kinetic energy:

$K = \frac{1}{2} I \omega^2$

- Angular momentum:

$L = I\omega$

Here:

$K = x, \quad L = y$

Step 2: Eliminate ω

From angular momentum:

$\omega = \frac{L}{I} = \frac{y}{I}$

Substitute into the kinetic energy:

$x = \frac{1}{2} I \left(\frac{y}{I}\right)^2$

$x = \frac{1}{2} I \cdot \frac{y^2}{I^2}$

$x = \frac{1}{2} \frac{y^2}{I}$

Step 3: Solve for I

$I = \frac{y^2}{2x}$

Final Answer:

The moment of inertia is

$$\boxed{\frac{y^2}{2x}}$$

✓ This corresponds to **Option B**.

Question23

Moment of inertia of a thin uniform rod rotating about the perpendicular axis passing through its centre is ' I '. If the same rod is bent in the form of ring, its moment of inertia about the diameter is ' I_1 '. If $\mathrm{I}_1 = \mathrm{xI}$, then the value of ' x ' is

Options:

A. $\frac{2\pi^2}{3}$

B. $\frac{3}{2\pi^2}$

C. $\frac{3\pi^2}{4}$

D. $\frac{4}{3\pi^2}$

Answer: B

Solution:

Step 1: Moment of inertia of thin uniform rod about perpendicular axis through its centre

- Length of rod = L, mass = M.
- For a thin rod about a perpendicular axis through its centre (i.e., an axis normal to rod at centre):

$I = \frac{1}{12}ML^2$

Step 2: Bend the same rod into the shape of a ring

- Now the rod of length L is bent into a ring (circle).
- Radius of the ring:

$2\pi R = L \quad \Rightarrow \quad R = \frac{L}{2\pi}$

- Mass = M.

Step 3: Moment of inertia of ring about its diameter

- For a thin ring of mass M, radius R:
- About axis through center and perpendicular to plane: MR^2.
- About diameter of ring (axis in plane, through centre): $\frac{1}{2}MR^2$.

$I_1 = \frac{1}{2}MR^2$

Step 4: Express I_1 in terms of given I

- From Step 1: $I = \frac{1}{12}ML^2$.
- From Step 2: $R = \frac{L}{2\pi}$.
- So:

$I_1 = \frac{1}{2}M\left(\frac{L}{2\pi}\right)^2 = \frac{1}{2}M \cdot \frac{L^2}{4\pi^2} = \frac{ML^2}{8\pi^2}$

Step 5: Ratio

$$\frac{I_1}{I} = \frac{\frac{ML^2}{8\pi^2}}{\frac{1}{12}ML^2} = \frac{12}{8\pi^2} = \frac{3}{2\pi^2}$$

Final Answer:

$$x = \frac{3}{2\pi^2}$$

✅ Correct option: **Option B**

Question24

A disc of mass 25 kg and radius 0.2 m is rotating at 240 r.p.m. A retarding torque brings it to rest in 20 second. If the torque is due to a force applied tangentially on the rim of the disc then the magnitude of the force is

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Options:

A. $\frac{\pi}{2}$ N

B. 2π N

C. πN

D. 4π N

Answer: C

Solution:

Given:

- Mass of disc, $m = 25\,\text{kg}$
- Radius, $R = 0.2\,\text{m}$
- Initial angular speed, $n = 240\,\text{r.p.m}$ (revolutions per minute)
- Time to stop, $t = 20\,\text{sec}$
- Final angular speed, $\omega_2 = 0$

- Tangential force F (to be found)

Step 1: Convert r.p.m. to angular velocity in rad/s

$\omega_1 = 2\pi n/60$

Substituting values:

$\omega_1 = 2\pi \times 240/60 = 2\pi \times 4 = 8\pi$ rad/s

Step 2: Find angular deceleration (α)

Using equation:

$\omega_2 = \omega_1 + \alpha t$

We have $\omega_2 = 0$, so:

$0 = 8\pi + \alpha \times 20 \implies \alpha = -\frac{8\pi}{20} = -\frac{2\pi}{5}$ rad/s^2

Step 3: Moment of inertia of a disc about its axis

$I = \frac{1}{2}mR^2$

Substitute:

$I = \frac{1}{2} \times 25 \times (0.2)^2 = \frac{1}{2} \times 25 \times 0.04 = 12.5 \times 0.04 = 0.5$ kg m^2

Step 4: Calculate retarding torque (τ)

$\tau = I \cdot \alpha$

$\tau = 0.5 \times \frac{2\pi}{5} = 0.5 \times \frac{2\pi}{5} = \frac{\pi}{5}$ N m

(Negative sign is for direction; magnitude only.)

Step 5: Relate torque to force

Torque produced by force applied at rim:

$\tau = F \times R \implies F = \frac{\tau}{R}$

$F = \frac{\frac{\pi}{5}}{0.2} = \frac{\pi}{5} \times \frac{1}{0.2} = \frac{\pi}{5} \times 5 = \pi$ N

Final Answer:

Option C

$\boxed{\pi \text{ N}}$

Question25

Two circular loops P and Q of radii ' r ' and ' nr ' are made respectively from a uniform wire. Moment of inertia of loop Q about its axis is 4 times that of loop P about its axis. The value of n is

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Options:

A. $(2)^{-2/3}$

B. $(2)^{2/3}$

C. $\sqrt{2}$

D. $2^{1/3}$

Answer: B

Solution:

Let the mass of wire used to make loop P be m_P and for loop Q be m_Q.

Since both loops are made from uniform wire of the **same length**, let's first find the relationship between their masses and radii.

Step 1: Equate lengths of wires

Length of wire in $P = 2\pi r$

Length of wire in $Q = 2\pi(nr)$

Since the wires are of equal length:

$2\pi r_P = 2\pi r_Q \implies r_P = r_Q$

But here, the loops are of radii r and nr, so the length of the wire for P is $2\pi r$, and for Q it is $2\pi nr$.

Let mass per unit length = λ.

So,

$m_P = \lambda \cdot 2\pi r$

$m_Q = \lambda \cdot 2\pi nr$

Step 2: Write the moments of inertia

The moment of inertia of a thin ring (loop) about its axis:

$I = mr^2$

So,

$I_P = m_P r^2 = (\lambda \cdot 2\pi r) \cdot r^2 = 2\pi\lambda r^3$

$I_Q = m_Q (nr)^2 = (\lambda \cdot 2\pi nr) \cdot (n^2 r^2) = 2\pi\lambda nr \cdot n^2 r^2 = 2\pi\lambda n^3 r^3$

Step 3: Use the condition $I_Q = 4I_P$

From above,

$I_Q = 2\pi\lambda n^3 r^3$

$I_P = 2\pi\lambda r^3$

Given,

$I_Q = 4I_P$

So,

$2\pi\lambda n^3 r^3 = 4 \cdot 2\pi\lambda r^3$

$n^3 = 4$

$n = 4^{1/3} = (2^2)^{1/3} = 2^{2/3}$

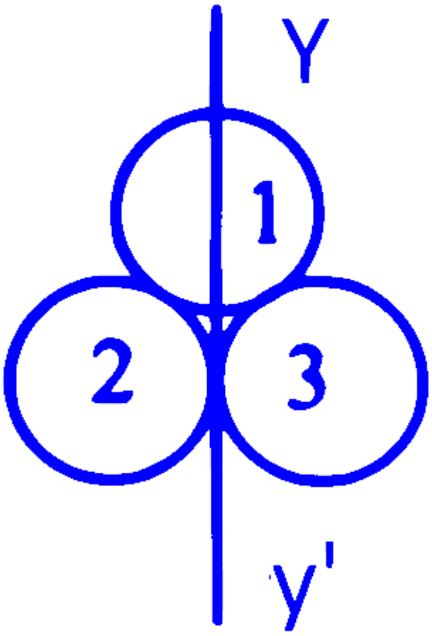
Final Answer:

$\boxed{(2)^{2/3}}$

So, the correct option is **Option B**.

Question26

Three spheres, each of mass ' m ' and radius ' r ' are placed as shown in figure. Consider an axis YY', which is touching two spheres and passing through the diameter of third sphere. The moment of inertia of the system consisting of these three spheres about YY' axis is



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Options:

A. $\frac{7}{5}\mathrm{mr}^2$

B. $\frac{2}{5}\mathrm{mr}^2$

C. $\frac{16}{5}\mathrm{mr}^2$

D. $\frac{\mathrm{mr}^2}{2}$

Answer: C

Solution:

Moment of inertia of a solid sphere $= \frac{2}{5}\mathrm{mr}^2$

Moment of inertia of the upper sphere $= \frac{2}{5}\mathrm{mr}^2$

For each lower sphere M.I. $= \frac{2}{5}\mathrm{mr}^2 + \mathrm{mr}^2$

$$= \frac{7}{5}\mathrm{mr}^2$$

Total moment of inertia $\mathrm{I}=\frac{2}{5}\mathrm{mr}^2+2\times\frac{7}{5}\mathrm{mr}^2$

$=\frac{16}{5}\mathrm{mr}^2$

Question27

A solid sphere rolling without friction on a horizontal surface with a constant speed of $2\ \mathrm{m/s}$, rolls up on an inclined ramp which is inclined at 30°. The maximum distance travelled by the sphere on the inclined ramp is (acceleration due to gravity $g=10\ \mathrm{m/s^2}, \sin 30^{\circ}=\frac{1}{2}$)

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Options:

A. 56 cm

B. 25 cm

C. 47 cm

D. 30 cm

Answer: A

Solution:

Sphere's Kinetic Energy = Translational energy + Rotational energy

$$=\frac{1}{2}mv^2+\frac{1}{5}mv^2$$
$$=\frac{7}{10}mv^2$$

At maximum height h ,

Total K.E. = Potential energy

$$\frac{7}{10}\mathrm{mv}^2=\mathrm{mgh}$$
$$\mathrm{h}=\frac{7\mathrm{v}^2}{10\ \mathrm{g}}=\frac{7\times 2^2}{10\times 10}=0.28\ \mathrm{m}$$

Distance along the incline

$d = \frac{h}{\sin\theta} = \frac{0.28}{\sin(30^\circ)} = 0.56 \text{ m} = 56 \text{ cm}$

Question28

A disc of mass ' m ' and radius ' r ' rolls down an inclined plane of height ' h '. When it reaches the bottom of the plane, its rotational kinetic energy is ($\mathrm{g}=$ acceleration due to gravity)

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Options:

A. $\frac{\mathrm{mgh}}{3}$

B. $\frac{\mathrm{mgh}}{6}$

C. $\frac{\mathrm{mgh}}{2}$

D. $\frac{\mathrm{mgh}}{4}$

Answer: A

Solution:

Total K.E. $= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$$= \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right) \frac{v^2}{R^2}$$
$$= \frac{1}{2} m v^2 + \frac{1}{4} m v^2 = \frac{3}{4} m v^2$$

At foot of the plane, total K.E. = P.E.

$$\therefore \quad \frac{3}{4} \mathrm{mv}^2 = \mathrm{mgh}$$
$$\Rightarrow \quad \mathrm{mv}^2 = \frac{4}{3} \mathrm{mgh}$$

But, rotational K.E. $= \frac{1}{4} m v^2 = \frac{m g h}{3}$

Question29

Two discs A and B of same material and thickness have radii R and $3R$ respectively. Their moments of inertia about their axis will be in the ratio

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Options:

A. 3 : 1

B. 1 : 9

C. 1 : 81

D. 1 : 27

Answer: C

Solution:

Let the thickness of both discs be t, and their material has density ρ.

Let disc A have radius R.

Let disc B have radius $3R$.

Step 1: Find Mass of Each Disc

Volume of a disc = Area × Thickness

For disc of radius r and thickness t:

Volume $= \pi r^2 t$

So, mass M of a disc:

$M = (\text{Volume}) \times (\text{Density}) = \pi r^2 t \rho$

For disc A:

$M_A = \pi R^2 t \rho$

For disc B (radius $3R$):

$M_B = \pi (3R)^2 t \rho = \pi \cdot 9R^2 t \rho = 9\pi R^2 t \rho$

Step 2: Moment of Inertia of a Uniform Disc about Its Axis

Formula:

$I = \frac{1}{2} M r^2$

For A:

$I_A = \frac{1}{2} M_A R^2 = \frac{1}{2}(\pi R^2 t \rho) R^2 = \frac{1}{2} \pi t \rho R^4$

For B:

$I_B = \frac{1}{2} M_B (3R)^2 = \frac{1}{2}(9 \pi R^2 t \rho) \times (9R^2)$

$I_B = \frac{1}{2} \cdot 9 \pi R^2 t \rho \cdot 9R^2 = \frac{1}{2} \cdot 81 \pi t \rho R^4 = \frac{81}{2} \pi t \rho R^4$

Step 3: Ratio of Moments of Inertia

$\frac{I_A}{I_B} = \frac{\frac{1}{2} \pi t \rho R^4}{\frac{81}{2} \pi t \rho R^4} = \frac{1}{81}$

So,

$I_A : I_B = 1 : 81$

Answer:

Option C $(1 : 81)$ is correct.

Question30

An inclined plane makes an angle 30° with the horizontal. A solid sphere rolling down an inclined plane from rest without slipping has linear acceleration ($\mathrm{g} =$ acceleration due gravity) ($\sin 30^{\circ} = 0.5$)

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Options:

A. $\frac{5 \mathrm{~g}}{7}$

B. $\frac{5 \mathrm{~g}}{14}$

C. $\frac{2 \mathrm{~g}}{3}$

D. $\frac{\mathrm{g}}{3}$

Answer: B

Solution:

For a solid sphere rolling down an inclined plane **without slipping**:

Let the **angle of inclination** be $\theta = 30^\circ$

Let the **linear acceleration** be a.

Let the **radius** of the sphere be R and its **mass** be m.

Step 1: Forces Acting on Sphere

1. Down the incline: $mg\sin\theta$ (component of weight)
2. Up the incline: Friction

Step 2: Equations of Motion

Translational motion (along incline):

$mg\sin\theta - f = ma$

Where f is the frictional force.

Rotational motion (about the centre of mass):

$fR = I\alpha$

Where I is the moment of inertia about its centre, and α is the angular acceleration.

For a **solid sphere**,

$I = \frac{2}{5}mR^2$

Step 3: Rolling Condition

Since the sphere rolls **without slipping**, velocity relation:

$a = R\alpha$

So,

$fR = I\alpha = I\left(\frac{a}{R}\right) = \frac{2}{5}mR^2 \cdot \frac{a}{R} = \frac{2}{5}mRa$

Or,

$f = \frac{2}{5}ma$

Step 4: Substitute f in the Translational Equation

Recall:

$mg\sin\theta - f = ma$

Plug in f:

$mg\sin\theta - \frac{2}{5}ma = ma$

So,

$mg \sin \theta = ma + \frac{2}{5}ma = ma\left(1 + \frac{2}{5}\right) = ma \cdot \frac{7}{5}$

Step 5: Solve for a

$mg \sin \theta = \frac{7}{5}ma$

$a = \frac{5}{7}g \sin \theta$

Step 6: Plug in $\sin 30^\circ = 0.5$

$a = \frac{5}{7}g \times 0.5 = \frac{5g}{14}$

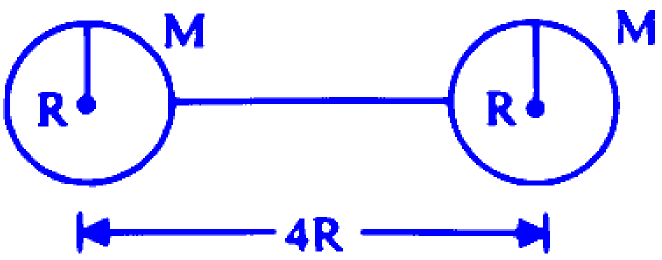
Final Answer

Option B is correct.

$\boxed{\frac{5g}{14}}$

Question31

Two spheres each of mass M and radius R are connected with a massless rod of length 4 R . The moment of inertia of the system about an axis passing through the centre of one of the spheres and perpendicular to the rod will be



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Options:

A. $\frac{21}{5}\mathrm{MR}^2$

B. $\frac{84}{5}\mathrm{MR}^2$

C. $\frac{42}{5}\mathrm{MR}^2$

D. $\frac{5}{21}\mathrm{MR}^2$

Answer: B

Solution:

From parallel axis theorem, $\mathrm{I_o} = \mathrm{I_c} + \mathrm{Mh}^2$ Let the moment of inertia of sphere 1 be

$I_1 = \frac{2}{5}M(R)^2 + M(4R)^2$

and,

Let the moment of inertia of sphere 2 be

$I_2 = \frac{2}{5}M(R)^2$

Moment of inertia of the rod $I_3 = 0$

$\therefore$ Moment of inertia of the system,

$$\begin{aligned} \mathrm{I} &= \mathrm{I_1} + \mathrm{I_2} + \mathrm{I_3} \\ \mathrm{I} &= \frac{2}{5}\mathrm{M(R)}^2 + \mathrm{M(4R)}^2 + \frac{2}{5}\mathrm{M(R)}^2 \\ &= \frac{4}{5}\mathrm{M(R)}^2 + 16\mathrm{MR}^2 \\ &= \frac{4}{5}\mathrm{MR}^2 + \frac{80}{5}\mathrm{MR}^2 = \frac{84}{5}\mathrm{MR}^2 \end{aligned}$$

Question32

A body slides down a smooth inclined plane of inclination θ and reaches the bottom with velocity V . If the same body is a ring which rolls down the same inclined plane then linear velocity at the bottom of plane is

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Options:

A. $\frac{\mathrm{V}}{\sqrt{2}}$

B. V

C. 2 V

D. $\frac{V}{2}$

Answer: A

Solution:

Let's solve step by step:

Step 1: Sliding Body (Smooth Incline, No Rotation)

- The body descends a height h, starting from rest.
- By law of conservation of energy:

$\text{Potential Energy at Top} = \text{Kinetic Energy at Bottom}$

$mgh = \frac{1}{2}mv^2$

- Solving for v:

$v = \sqrt{2gh}$

- You are given that this velocity is V.

$V = \sqrt{2gh}$

Step 2: Rolling Ring (No Slipping, Both Translational and Rotational Kinetic Energy)

- For a rolling ring:
- Mass = m,
- Velocity at bottom = v,
- Moment of inertia about centre = $I = mR^2$,
- Angular velocity $= \omega = \frac{v}{R}$.
- Law of conservation of energy:

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$mgh = mv^2$

$v^2 = gh$

$v = \sqrt{gh}$

Step 3: Relate to V

$V = \sqrt{2gh} \implies \sqrt{gh} = \frac{V}{\sqrt{2}}$

So, for the rolling ring, the linear velocity at the bottom is:

$$\boxed{\frac{V}{\sqrt{2}}}$$

Correct Option: (A) $\frac{V}{\sqrt{2}}$

Question33

If $\vec{F} = (5\hat{\imath} - 10\hat{\jmath})$ and $\vec{r} = (4\hat{\imath} - 3\hat{\jmath})$, then the torque acting on the object will be

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Options:

A. $\hat{\text{i}} - 2\hat{\text{j}}$

B. $2\hat{\text{i}} - \hat{\text{j}}$

C. $25\hat{\text{k}}$

D. $-25\hat{\text{k}}$

Answer: D

Solution:

Given:

$\vec{F} = 5\hat{\imath} - 10\hat{\jmath}$

$\vec{r} = 4\hat{\imath} - 3\hat{\jmath}$

Step 1:

Torque $(\vec{\tau})$ is given by the cross product:

$\vec{\tau} = \vec{r} \times \vec{F}$

Step 2:

Write vectors in component form:

$\vec{r} = 4\hat{i} + (-3)\hat{j} + 0\hat{k}$

$\vec{F} = 5\hat{i} + (-10)\hat{j} + 0\hat{k}$

Step 3:

Use the determinant method for cross product:

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 0 \\ 5 & -10 & 0 \end{vmatrix}$$

Step 4:

Expand the determinant:

$$\vec{\tau} = \hat{i}\begin{vmatrix} -3 & 0 \\ -10 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 4 & 0 \\ 5 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 4 & -3 \\ 5 & -10 \end{vmatrix}$$

Calculate each minor:

- $\begin{vmatrix} -3 & 0 \\ -10 & 0 \end{vmatrix} = (-3)(0) - (0)(-10) = 0$
- $\begin{vmatrix} 4 & 0 \\ 5 & 0 \end{vmatrix} = (4)(0) - (0)(5) = 0$
- $\begin{vmatrix} 4 & -3 \\ 5 & -10 \end{vmatrix} = (4)(-10) - (-3)(5) = -40 + 15 = -25$

So,

$\vec{\tau} = 0\hat{i} - 0\hat{j} - 25\hat{k}$

$\vec{\tau} = -25\hat{k}$

Final Answer:

$\boxed{-25\hat{k}}$

So, the correct option is **Option D**.

Question34

Four particles each of mass M are placed at the corners of a square of side L. The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is

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Options:

A. L

B. $\frac{L}{2}$

C. $\frac{L}{4}$

D. $\frac{L}{\sqrt{2}}$

Answer: D

Solution:

Let the four particles each of mass M be placed at the corners of a square of side L.

Step 1: Calculate the distance of each particle from the centre

- The centre of the square is equidistant from all corners.
- The distance from the centre to a corner (let's call it r):

$r =$ half the diagonal of the square $= \frac{L}{\sqrt{2}}$

Step 2: Calculate the moment of inertia about the given axis

The axis is perpendicular to the square and passes through its centre.

- Moment of inertia for each mass about the axis:

$$I_{\text{each}} = Mr^2 = M\left(\frac{L}{\sqrt{2}}\right)^2 = M \cdot \frac{L^2}{2}$$

- For 4 masses:

$$I_{\text{total}} = 4 \cdot \frac{ML^2}{2} = 2ML^2$$

Step 3: Total mass of the system

$$M_{\text{total}} = 4M$$

Step 4: Formula for radius of gyration K

The moment of inertia is also given by: $I = M_{\text{total}} \ K^2$

So,

$$I_{\text{total}} = M_{\text{total}} \ K^2 \implies 2ML^2 = 4MK^2$$

Step 5: Solve for K

$$4MK^2 = 2ML^2K^2 = \frac{2ML^2}{4M}K^2 = \frac{L^2}{2}K = \frac{L}{\sqrt{2}}$$

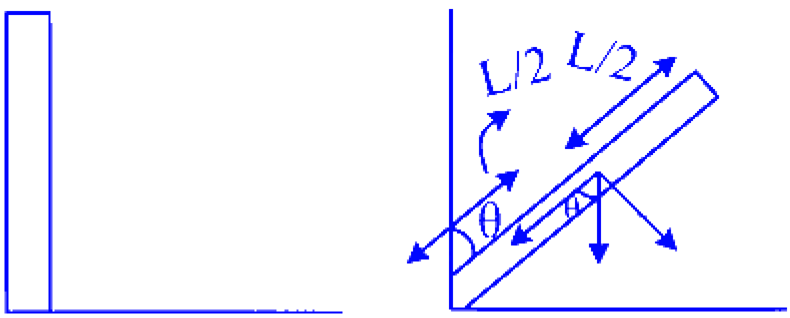
Correct option:

$$\boxed{\frac{L}{\sqrt{2}}}$$

So, the answer is **Option D**.

Question35

A thin uniform rod of mass ' m ' and length ' L ' is pivoted at one end so that it can rotate in a vertical plane. The free end is held vertically above pivot and then released. The angular acceleration of the rod when it makes an angle ' θ ' with the vertical is [consider negligible friction at the pivot] ($\mathrm{g}=$ acceleration due to gravity)



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Options:

A. $\frac{3 g \sin \theta}{2 L}$

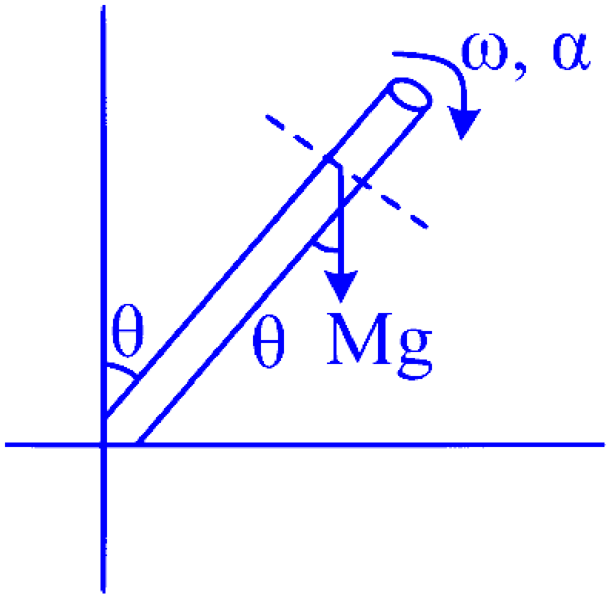
B. $\frac{3 g \cos \theta}{2 L}$

C. $\frac{2 g \sin \theta}{3 L}$

D. $\frac{2 g \cos \theta}{3 L}$

Answer: A

Solution:



Torque at angle θ,

$\tau = Mg\sin\theta\frac{L}{2}$

Also, we know , $\tau = \mathrm{I}\alpha$

$\Rightarrow \quad I\alpha = mg\sin\theta\frac{L}{2}$

$\because$ For a thin uniform rod, the moment of inertia about an axis perpendicular to its length and passing through its one end is $\frac{\mathrm{ML}^2}{3}$, the above equation becomes:

$$\Rightarrow \quad \frac{\mathrm{ML}^2}{3}\cdot\alpha = \mathrm{Mg}\sin\theta\frac{\mathrm{L}}{2}$$
$$\Rightarrow \quad \frac{\mathrm{L}\alpha}{2} = \mathrm{g}\frac{\sin\theta}{2}$$
$$\Rightarrow \quad \alpha = \frac{3\,\mathrm{g}\sin\theta}{2\,\mathrm{L}}$$

Question36

Three point masses, each of mass ' m ' are placed at the corners of an equilateral triangle of side ' L '. The moment of inertia of the system about an axis passing through one of the vertices and parallel to the side joining other two vertices will be

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Options:

A. $\frac{3\ mL^2}{4}$

B. $\frac{mL^2}{4}$

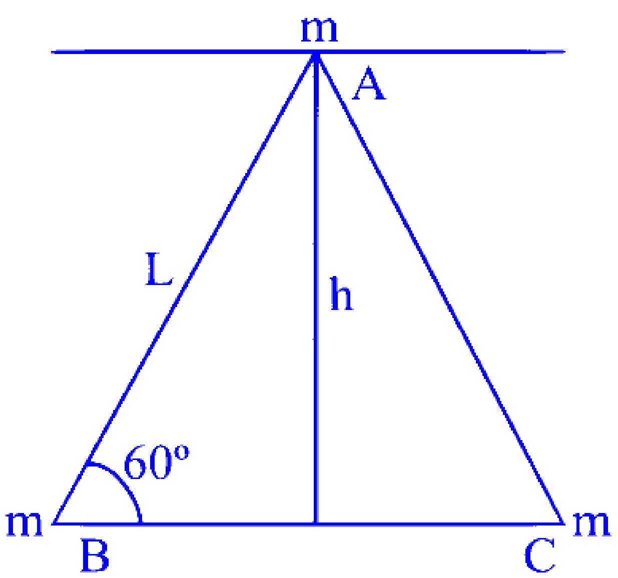
C. $\frac{3\ mL^2}{2}$

D. $\frac{mL^2}{2}$

Answer: C

Solution:

Consider the mass at vertex A as shown in figure.



Hence, M.I. about the line through A is,

$I = 2mh^2$

From figure,

$$h = L\sin 60^\circ = L \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad I = 2\ m \times \frac{3}{4} L^2 = \frac{3}{2}\ mL^2$$

Question37

Two spheres of equal masses, one of which is a thin spherical shell and the other solid sphere, have the same moment of inertia about their respective diameters. The ratio of their radii is

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Options:

A. $3 : 5$

B. $\sqrt{3} : \sqrt{5}$

C. $\sqrt{3} : \sqrt{7}$

D. $5 : 7$

Answer: B

Solution:

Let the radii of the thin spherical shell and the solid sphere be R_1 and R_2, respectively. Then, the moment of inertia of the shell about its diameter is given by,

$I_{\text{shell}} = \frac{2}{3} MR_1^2$ .... (i)

And the moment of inertia of the solid sphere is given by,

$I_{\text{sphere}} = \frac{2}{5} MR_2^2$ ...(ii)

Given that, the masses and moment of inertia for both the bodies are equal, then from equations (i) and (ii),

$$\frac{2}{3} MR_1^2 = \frac{2}{5} MR_2^2$$

$$\therefore \quad \frac{R_1^2}{R_2^2} = \frac{3}{5}$$

$$\therefore \quad R_1 : R_2 = \sqrt{3} : \sqrt{5}$$

Question38

Two loops P and Q of radii R_1 and R_2 are made from uniform metal wire of same material. I_p and I_Q be the moment of inertia of loop P and Q respectively then ratio R_1/R_2 is (Given $I_P/I_Q = 27$)

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Options:

A. 4 : 1

B. 3 : 1

C. 9 : 1

D. 6 : 1

Answer: B

Solution:

Two loops made from same material

$\mathrm{I} = \mathrm{MR}^2$ ...(i)

$\mathrm{M} = \sigma.2\pi\mathrm{R}$ ...(σ is mass per unit length)

Substituting in (i),

$\mathrm{I} = \sigma.2\pi\mathrm{R}^3$ .... (ii)

$\therefore \quad \frac{\mathrm{I_P}}{\mathrm{I_Q}} = \frac{\sigma \cdot 2\pi\mathrm{R}_1^3}{\sigma \cdot 2\pi\mathrm{R}_2^3}$ ...[From(ii)]

$$\frac{\mathrm{I_P}}{\mathrm{I_Q}} = \left(\frac{\mathrm{R}_1}{\mathrm{R}_2}\right)^3$$

$$\frac{27}{1} = \left(\frac{\mathrm{R}_1}{\mathrm{R}_2}\right)^3 \quad \ldots\left(\text{given, } \frac{\mathrm{I_P}}{\mathrm{I_Q}} = 27\right)$$

$$\therefore \quad \frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{3}{1}$$

Question39

A body of mass m slides down an incline and reaches the bottom with a velocity V . If the same mass were in the form of a disc which rolls down this incline, the velocity of the disc at bottom would have been

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Options:

A. $\mathrm{v} \sqrt{\frac{3}{4}}$

B. $\mathrm{v} \sqrt{\frac{3}{2}}$

C. $v \sqrt{\frac{1}{3}}$

D. $\mathrm{v} \sqrt{\frac{2}{3}}$

Answer: D

Solution:

When a body of mass m slides down an incline without friction, it converts all its potential energy at the top into translational kinetic energy at the bottom. The velocity V at the bottom in this case is given by:

$V=\sqrt{2 g h}$

where g is the acceleration due to gravity, and h is the height of the incline.

For a disc that rolls without slipping, the energy at the bottom is distributed between translational kinetic energy and rotational kinetic energy. The total mechanical energy at the top is the same potential energy, but at the bottom, it will be divided as:

Translational kinetic energy: $\frac{1}{2} m v^2$

Rotational kinetic energy: $\frac{1}{2}I\omega^2$

For a disc, the moment of inertia I is $\frac{1}{2}mr^2$, and the relationship between linear velocity v and angular velocity ω is $\omega = \frac{v}{r}$.

Substituting these into the equations for kinetic energy, the total kinetic energy at the bottom becomes:

$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$

The combined kinetic energy thus equals:

$\frac{3}{4}mv^2$

Equating the initial potential energy to the total kinetic energy at the bottom:

$mgh = \frac{3}{4}mv^2$

Solving for v, we find:

$gh = \frac{3}{4}v^2$

$v^2 = \frac{4}{3}gh$

$v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}} \cdot \sqrt{gh}$

Since $\sqrt{gh} = \frac{V}{\sqrt{2}}$, the velocity v of the disc at the bottom becomes:

$v = \sqrt{\frac{4}{3}} \cdot \frac{V}{\sqrt{2}}$

Further simplification gives the final expression:

$v = \frac{V}{\sqrt{3/2}} = V \cdot \sqrt{\frac{2}{3}}$

Therefore, the correct option is **Option D:** $v\sqrt{\frac{2}{3}}$.

Question40

The radius of gyration of a circular disc of radius R and mass m rotating about diameter as axis is

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Options:

A. R$\sqrt{2}$

B. R/$\sqrt{2}$

C. R/2

D. R

Answer: C

Solution:

To determine the radius of gyration of a circular disc of radius R and mass m rotating about its diameter, use the formula for the radius of gyration:

$$k = \sqrt{\frac{I}{m}}$$

where I is the moment of inertia of the disc about the diameter, and m is the mass of the disc.

For a circular disc of mass m and radius R, the moment of inertia I about its diameter is given by:

$$I = \frac{1}{4}mR^2$$

Substituting the expression for I into the formula for the radius of gyration:

$$k = \sqrt{\frac{\frac{1}{4}mR^2}{m}} = \sqrt{\frac{1}{4}R^2} = \frac{R}{2}$$

Therefore, the radius of gyration of the circular disc about its diameter is

Option C: $\frac{R}{2}$

Question41

A thin uniform metal rod of mass ' M ' and length ' L ' is swinging about a horizontal axis passing through its end. Its maximum angular velocity is ' ω '. Its centre of mass rises to a maximum height of (g = Acceleration due to gravity)

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Options:

A. $\frac{L^2\omega^2}{3\ g}$

B. $\frac{L^2\omega^2}{2\ g}$

C. $\frac{L^2\omega^2}{6\ g}$

D. $\frac{L^2\omega^2}{4\ g}$

Answer: C

Solution:

By conservation of energy, $\frac{1}{2}I\omega^2 = Mgh$

$\therefore \quad h = \frac{I\omega^2}{2Mg} = h \quad .... (i)$

The M.I of a uniform rod about an axis passing through its centre is $\frac{ML^2}{12}$.

As the axis passing through the end, using parallel axis theorem,

$I = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3} \quad .... (ii)$

Putting (ii) into (i),

$h = \frac{L^2\omega^2}{6g}$

Question42

Three thin rods, each of mass ' M ' and length ' L ' are placed along X, Y and Z axes which are mutually perpendicular. One end of each rod is at origin. M. I. of the system about Z axis is

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Options:

A. $\frac{3 \mathrm{ML}^2}{4}$

B. $\frac{2 M^2}{5}$

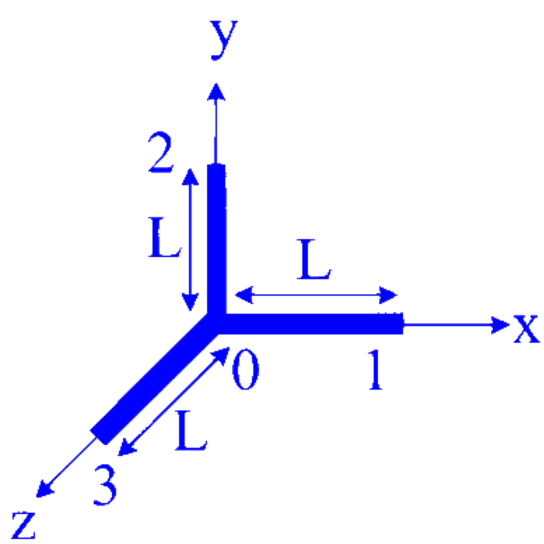
C. $\frac{2 \mathrm{ML}^2}{3}$

D. $\frac{3 \mathrm{ML}^2}{5}$

Answer: C

Solution:

$$\begin{aligned} & \mathrm{I}=\mathrm{I}_1+\mathrm{I}_2+\mathrm{I}_3 \\ \therefore \quad & \mathrm{I}=\frac{\mathrm{ML}^2}{3}+\frac{\mathrm{ML}^2}{3}+0 \\ \therefore \quad & \mathrm{I}=\frac{2 \mathrm{ML}^2}{3} \end{aligned}$$



Question43

If the angular velocity of a body rotating about the given axis increases by 20%, then its kinetic energy of rotation will increase by

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Options:

A. 20%

B. 30%

C. 44%

D. 66%

Answer: C

Solution:

$\omega_2 = \frac{120}{100}\omega_1 = 1.2\omega_1$

$$\frac{KE_2}{KE_1} = \frac{\frac{1}{2}\mathrm{I}\omega_2^2}{\frac{1}{2}\mathrm{I}\omega_1^2} = \frac{\omega_2^2}{\omega_1^2} = \frac{(1.2\omega_1)^2}{\omega_1^2} = 1.44$$

$$\therefore \quad \mathrm{KE}_2 = 1.44\mathrm{KE}_1$$
$$\therefore \quad \left(\frac{\mathrm{KE}_2 - \mathrm{KE}_1}{\mathrm{KE}_1}\right) \times 100 = \left(\frac{1.44\mathrm{KE}_1 - \mathrm{KE}_1}{\mathrm{KE}_1}\right) \times 100$$
$$= 0.44 \times 100 = 44\%$$

Question44

Four thin metal rods each of mass ' M ' and length ' L ', are welded end to end to form a square. The moment of inertia of the system about an axis passing through the centre of the square and perpendicular to its plane is

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Options:

A. $\frac{\mathrm{ML}^2}{3}$

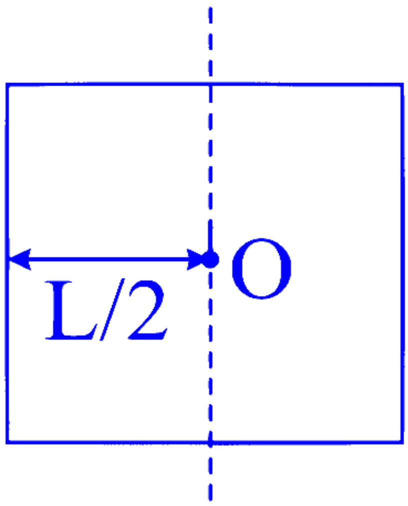
B. $\frac{2\mathrm{ML}^2}{3}$

C. $\frac{2ML^2}{9}$

D. $\frac{4ML^2}{3}$

Answer: D

Solution:



Moment of inertia of the system about an axis passing through O

$$\begin{aligned} I_0 &= 4\left[\text{MI of rod} + m\left(\frac{L}{2}\right)^2\right] \\ &= 4\left[\frac{ML^2}{12} + \frac{ML^2}{4}\right] \\ &= \frac{4}{3}ML^2 \end{aligned}$$

Question45

Moment of inertia of a disc about an axis passing through its centre and perpendicular to its plane is 'I'. The ratio of moment of inertia about a parallel axis tangential to its rim to passing through a point midway between the centre and the rim is

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Options:

A. 2 : 1

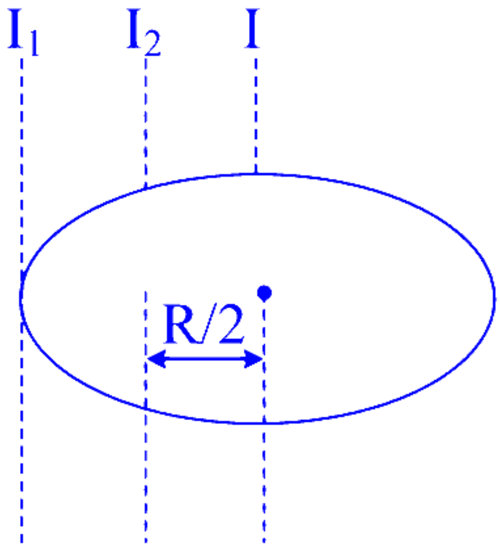
B. 3 : 1

C. 4 : 1

D. 6 : 1

Answer: A

Solution:



$I = \frac{MR^2}{2} \Rightarrow$ M.I. of disc about an axis passing through centre of mass

By parallel axis theorem,

M.I for axis tangential to rim,

$I_1 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

M.I for axis passing through a point midway between centre and rim ($h = R/2$),

$$I_2 = \frac{MR^2}{2} + \frac{MR^2}{4} = \frac{3}{4}MR^2$$
$$\frac{I_1}{I_2} = \frac{\frac{3}{2}MR^2}{\frac{3}{4}MR^2} = \frac{2}{1} \Rightarrow 2 : 1$$

Question46

An inclined plane makes an angle 30° with horizontal. A solid sphere rolls down from the top of the inclined plane from rest without slipping has a linear acceleration along the plane equal to (where g is acceleration due to gravity) (given $\sin 30^{\circ}=0.5$)

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Options:

A. $\frac{5g}{14}$

B. $\frac{5g}{4}$

C. $\frac{2g}{3}$

D. $\frac{g}{3}$

Answer: A

Solution:

$$a=\frac{g \sin \theta}{\left(1+\frac{K^2}{R^2}\right)}=\frac{g \sin 30^{\circ}}{\left(1+\frac{2}{5}\right)}$$

$$\therefore \quad a=\frac{5g}{7} \times\left(\frac{1}{2}\right)=\frac{5g}{14}$$

Question47

Two bodies A and B have their moments of inertia I_1 and I_2 respectively about their axis of rotation. If their kinetic energies of rotation are equal and their angular momenta L_1 and L_2 respectively are in the ratio $1:\sqrt{3}$, then I_2 will be

Options:

A. $\frac{1}{3}\mathrm{I}_1$

B. $\sqrt{3}I_1$

C. $2\mathrm{I}_1$

D. $3\mathrm{I}_1$

Answer: D

Solution:

$$(\text{K.E.})_A = (\text{K.E.})_B$$

$$\frac{1}{2}\mathrm{I}_1\omega_1^2 = \frac{1}{2}\mathrm{I}_2\omega_2^2$$

$$\therefore \quad \frac{\omega_2^2}{\omega_1^2} = \frac{I_1}{I_2} \quad \text{.... (i)}$$

$$\text{Also, K.E} = \frac{1}{2}\mathrm{L}\omega$$

$$\therefore \quad \frac{1}{2}\mathrm{L}_1\omega_1 = \frac{1}{2}\mathrm{L}_2\omega_2$$

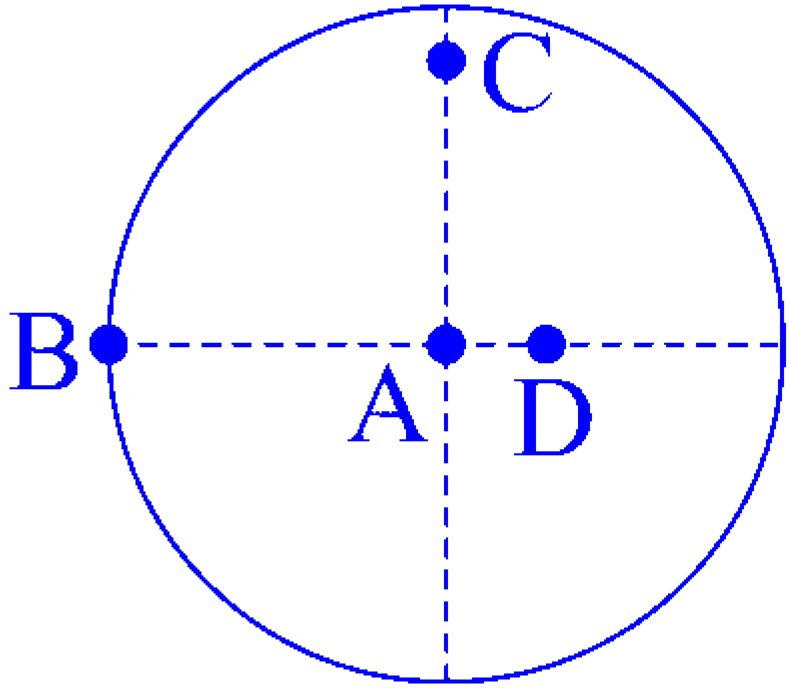
$$\therefore \quad \frac{\mathrm{L}_1}{\mathrm{L}_2} = \frac{\omega_2}{\omega_1} = \frac{1}{\sqrt{3}} \quad \text{.... (ii)}$$

$$\therefore \quad \frac{\mathrm{I}_1}{\mathrm{I}_2} = \frac{1}{3} \quad \text{... [From (i) and (ii)]}$$

$$\therefore \quad \mathrm{I}_2 = 3\mathrm{I}_1$$

Question48

The moment of inertia of uniform circular disc is maximum about an axis perpendicular to the disc and passing through point



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Options:

A. A

B. B

C. C

D. D

Answer: B

Solution:

Moment of Inertia $(\mathrm{I})=\mathrm{I}_{\mathrm{CM}}+\mathrm{Md}^2$

where d is distance of axis from COM

The I_{CM} and M is same for all the points and only ' d ' is varying

As axis B is having more distance from COM and thus will have more moment of inertia.

Question49

A ring and a disc roll on horizontal surface without slipping with same linear velocity. If both have same mass and total kinetic energy of the ring is 6 J then total kinetic energy of the disc is

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Options:

A. $\frac{3}{2}$ J

B. $\frac{5}{2}$ J

C. $\frac{7}{2}$ J

D. $\frac{9}{2}$ J

Answer: D

Solution:

Total $(\mathrm{K.E})_{\text{ring}} = \mathrm{Mv}^2$ .... (i)

Total (K.E) $)_{\text{disc}} = \frac{3}{4}\mathrm{Mv}^2$ .... (ii)

Dividing equation (ii) by equation (i)

$$\frac{(\mathrm{K.E})_{\text{disc}}}{(\mathrm{K.E})_{\text{ring}}} = \frac{\frac{3}{4}\mathrm{Mv}^2}{\mathrm{Mv}^2}$$

$$\therefore \quad (\mathrm{K.E})_{\text{disc}} = (\mathrm{K.E})_{\text{ring}} \times \frac{3}{4} = 6 \times \frac{3}{4} = \frac{9}{2}\ \mathrm{J}$$

Question50

A solid metallic sphere of radius ' R ' having moment of inertia 'I' about diameter is melted and recast into a solid disc of radius ' r ' of a uniform thickness. The moment of inertia of a disc about an axis passing through its edge and perpendicular to its plane is also equal to 'I'. The ratio $\frac{r}{R}$ is

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Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{2}{\sqrt{10}}$

D. $\frac{2}{\sqrt{15}}$

Answer: D

Solution:

M.I. of the solid sphere about a diameter

$\mathrm{I}=\frac{2}{5} \mathrm{MR}^2$ .... (i)

M.I. of the disc about an axis through its edge and perpendicular to its plane is

$\mathrm{I}=\frac{3}{2} \mathrm{Mr}^2$ ... (ii)

$\therefore \quad \frac{2}{5} M R^2=\frac{3}{2} M r^2$ .... [From (i) and (ii)]

$\therefore \quad r=\frac{2}{\sqrt{15}} \mathrm{R}$

Question51

A particle of mass ' m ' is rotating in a circular path of radius ' r '. Its angular momentum is ' L ' The centripetal force acting on it is ' F '. The relation between ' F ', ' L ', ' r ' and ' m ' is

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Options:

A. $\mathrm{F}=\frac{\mathrm{L}}{\mathrm{mr}^2}$

B. $\mathrm{L}=\mathrm{m}^2 \mathrm{Fr}^2$

C. $\frac{L^2}{m} = Fr^3$

D. $\frac{F}{L^3} = mr^2$

Answer: C

Solution:

The correct relationship between the centripetal force F, angular momentum L, radius r, and mass m can be derived using the concepts of circular motion and angular momentum.

Angular momentum L is given by:

$L = mvr$

where v is the tangential velocity. For an object in circular motion, the centripetal force F is also defined as:

$F = \frac{mv^2}{r}$

By rearranging the first equation for velocity, we have:

$v = \frac{L}{mr}$

Substitute v into the expression for F:

$F = \frac{m\left(\frac{L}{mr}\right)^2}{r} = \frac{m \cdot \frac{L^2}{m^2r^2}}{r}$

Simplifying further:

$F = \frac{L^2}{mr^3}$

Thus, the correct relationship is:

$\frac{L^2}{m} = Fr^3$

Therefore, the option that matches this expression is **Option C**:

$\frac{L^2}{m} = Fr^3$

Question52

Three thin rods, each mass ' 2 M ' and length ' L ' are placed along x, y and z axis which are mutually perpendicular. One end of each rod is at origin. Moment of inertia of the system about x - axis is

Options:

A. $\frac{4ML^2}{3}$

B. $\frac{ML^2}{12}$

C. $\frac{ML^2}{6}$

D. $\frac{2ML^2}{3}$

Answer: A

Solution:

Total moment of inertia,
$I_{total} = I_x + I_y + I_z$ .... (i)

Moment of inertia of thin rod, when axis is passing through one end and perpendicular to the rod,

$I = \frac{ML^2}{3}$ .... (ii)

$I_y = \frac{M_y L^2}{3} = \frac{2ML^2}{3}$ .... [From (i)]

$\ldots$ (given, $M_y = 2M$)

Similarly,

$I_2 = \frac{M_z L^2}{3} = \frac{2ML^2}{3}$ .... [From (ii)]

$\ldots$ (given, $M_z = 2M$)

$I_x = 0$ $\ldots$ ($\because$ Rod lies along X-axis)
Substituting in (i),

$\therefore$ $I_{total} = \frac{4ML^2}{3}$

Question53

A thin uniform rod of length ' L ' and mass ' M ' is swinging freely along a horizontal axis passing through its centre. Its maximum angular speed is ' ω '. Its centre of mass rises to a maximum height of [g = gravitational acceleration]

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Options:

A. $\frac{\omega^2 \mathrm{~L}^2}{12 \mathrm{~g}^2}$

B. $\frac{\omega^2 L^2 g}{6}$

C. $\frac{\omega^2 g}{12 \mathrm{~L}^2}$

D. $\frac{\omega^2 L^2}{24 \mathrm{~g}}$

Answer: D

Solution:

To determine the maximum height to which the center of mass of the swinging rod rises, we need to apply the principle of conservation of energy.

Initially, the entire energy of the system is kinetic because the rod is swinging with its maximum angular speed ω around its center. The moment of inertia I of a thin uniform rod of length L about an axis through its center is given by:

$I=\frac{1}{12} M L^2$

The initial kinetic energy (K_i) of the rod is thus:

$K_i=\frac{1}{2} I \omega^2=\frac{1}{2}\left(\frac{1}{12} M L^2\right) \omega^2=\frac{1}{24} M L^2 \omega^2$

At the highest point of the swing, all this kinetic energy is converted to potential energy (U_f), assuming no energy is lost:

$U_f=M g h$

where h is the maximum height attained by the center of mass and g is the gravitational acceleration. By conservation of energy:

$K_i=U_f$

$\frac{1}{24} M L^2 \omega^2=M g h$

Solving for h, we get:

$h=\frac{1}{24} \frac{L^2 \omega^2}{g}$

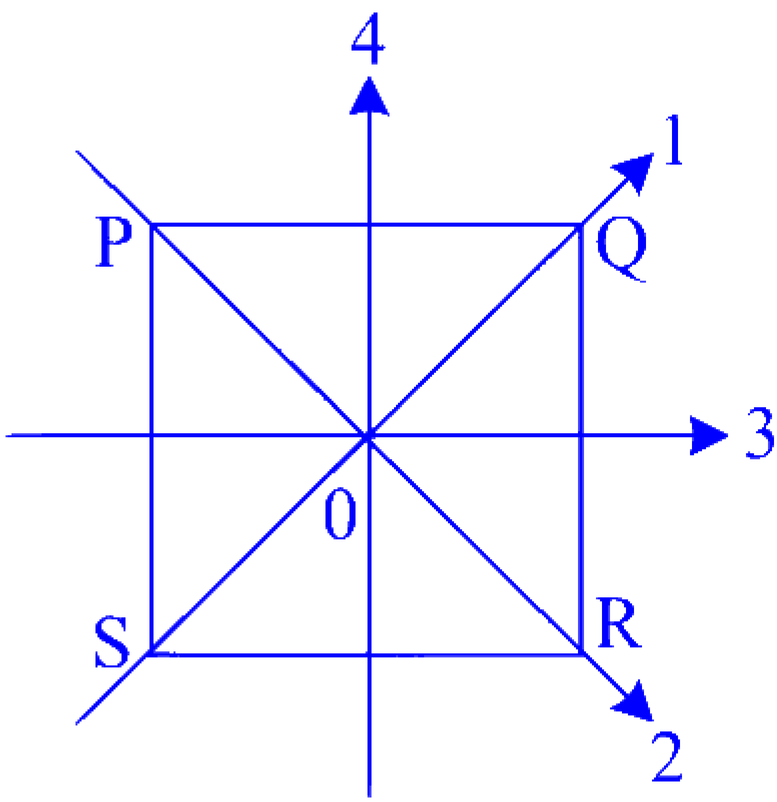
Thus, the maximum height of the center of mass is:

Option D:

$\frac{\omega^2 L^2}{24 \text{ g}}$

Question54

The moment of inertia of thin square plate PQRS of uniform thickness, about an axis passing through centre ' O ' and perpendicular to the plane of the plate is $(\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3, \mathrm{I}_4$ are respectively the moments of inertia about axis $1, 2, 3, 4$ which are in the plane of the plate as shown in figure)



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Options:

A. $\mathrm{I}_1+\mathrm{I}_2+\mathrm{I}_3$

B. $\mathrm{I}_1+\mathrm{I}_3+\mathrm{I}_4$

C. $\mathrm{I}_1+\mathrm{I}_2+\mathrm{I}_3+\mathrm{I}_4$

D. $\mathrm{I}_1+\mathrm{I}_3$

Answer: D

Solution:

Axis of I_1 and I_2 and that of I_3 and I_4 are pependicular to each other.

By theorem of perpendicular axis,

$\mathrm{I}=\mathrm{I}_1+\mathrm{I}_2 \quad$ or $\mathrm{I}=\mathrm{I}_3+\mathrm{I}_4 \quad \ldots .(\mathrm{i})$

As it is a square,

$\mathrm{I}_1=\mathrm{I}_2$ and $\mathrm{I}_3=\mathrm{I}_4$

From (i),

$$
\begin{aligned}
& \mathrm{I}_1=\mathrm{I}_2=\frac{\mathrm{I}}{2} \\
& \mathrm{I}_3=\mathrm{I}_4=\frac{\mathrm{I}}{2} \\
\therefore \quad & \mathrm{I}_3=\mathrm{I}_1 \\
\therefore \quad & \text{Momentof inertia of the plate }=\mathrm{I}=\mathrm{I}_1+\mathrm{I}_3
\end{aligned}
$$

Question55

A circular disc of radius ' R ' and thickness $\frac{R}{8}$ has moment of inertia 'I' about an axis passing through its centre and perpendicular to its plane. It is melted and recasted into a solid sphere then moment of inertia of sphere about an axis passing through diameter is

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Options:

A. I

B. $\frac{21}{3}$

C. $\frac{\mathrm{I}}{5}$

D. $\frac{\mathrm{I}}{10}$

Answer: C

Solution:

To solve the problem of finding the moment of inertia of a solid sphere formed by recasting a circular disc, first, consider the moment of inertia of the disc.

The moment of inertia I of a circular disc of radius R and mass m about an axis through its center and perpendicular to the plane of the disc is given by:

$I = \frac{1}{2}mR^2$

The volume of the disc is:

$V_{\text{disc}} = \pi R^2 \cdot \frac{R}{8} = \frac{\pi R^3}{8}$

Assuming the density is uniform, when the disc is melted into a solid sphere, the volume of the sphere is equal to the volume of the original disc:

$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{\pi R^3}{8}$

Solving for the radius r of the sphere:

$\frac{4}{3}\pi r^3 = \frac{\pi R^3}{8}$

Cancelling π and solving for r^3:

$r^3 = \frac{R^3}{32}$

Thus,

$r = \left(\frac{R^3}{32}\right)^{1/3} = \frac{R}{2}$

The moment of inertia I_{sphere} of a solid sphere of mass m and radius r about its diameter is:

$I_{\text{sphere}} = \frac{2}{5}mr^2$

Substitute the radius $r = \frac{R}{2}$:

$I_{\text{sphere}} = \frac{2}{5}m\left(\frac{R}{2}\right)^2 = \frac{2}{5}m\frac{R^2}{4} = \frac{mR^2}{10}$

We earlier established $mR^2 = 2I$, where I is the moment of inertia of the disc. Plug $mR^2 = 2I$ into the expression:

$I_{\text{sphere}} = \frac{(2I)}{10} = \frac{I}{5}$

Thus, the correct option is:

Option C: $\frac{I}{5}$

Question56

Two solid spheres (A and B) are made of metals having densities ρ_A and ρ_B respectively. If there masses are equal then ratio of their

moments of inertia $\left(\frac{I_B}{I_A}\right)$ about their respective diameter is

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Options:

A. $\left(\frac{\rho_B}{\rho_A}\right)^{2/3}$

B. $\left(\frac{\rho_A}{\rho_B}\right)^{2/3}$

C. $\frac{\rho_A}{\rho_B}$

D. $\frac{\rho_B}{\rho_A}$

Answer: B

Solution:

Mass = Volume × Density

Mass of spheres A and B is,

$M_A = \frac{4}{3}\pi R_A^3 \rho_A$ and $M_B = \frac{4}{3}\pi R_B^3 \rho_B$

If masses are equal,

$M_A = M_B$

$$\therefore \quad \frac{4}{3}\pi R_A^3 \rho_A = \frac{4}{3}\pi R_B^3 \rho_B$$

$$\frac{R_B}{R_A} = \left(\frac{\rho_A}{\rho_B}\right)^{\frac{1}{3}} \quad \ldots\ldots \text{(i)}$$

Moment of inertia of sphere about its diameter is given by, $I = \frac{2}{5}MR^2$

$I_A = \frac{2}{5}M_A R_A^2$ and $I_B = \frac{2}{5}M_B R_B^2$

$$\therefore \quad \frac{I_B}{I_A} = \frac{R_B^2}{R_A^2}$$

$$\frac{I_B}{I_A} = \left(\frac{\rho_A}{\rho_B}\right)^{\frac{2}{3}} \quad \ldots\ldots \text{[From (i)]}$$

Question57

A thin uniform circular disc of mass ' M ' and radius ' R ' is rotating with angular velocity ' ω ' in a horizontal plane about an axis passing through its centre and perpendicular to its plane.Another disc of same radius but of mass $\left(\frac{\mathrm{M}}{3}\right)$ is placed gently on the first disc co-axially. The new angular velocity will be

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Options:

A. $\frac{2}{3} \omega$

B. $\frac{3}{4} \omega$

C. $\frac{4}{3} \omega$

D. $\frac{5}{4} \omega$

Answer: B

Solution:

Angular momentum $=\mathrm{I} \omega$

By conservation of angular momentum,

$$\begin{aligned} & \mathrm{I}_1 \omega_1=\mathrm{I}_2 \omega_2 \\ & \text { Here, } \mathrm{I}_1=\frac{\mathrm{MR}^2}{2}, \mathrm{I}_2=\frac{(\mathrm{M}+\mathrm{M} / 3)}{2} \mathrm{R}^2=\frac{2 \mathrm{MR}^2}{3} \\ \therefore \quad & \frac{\mathrm{MR}^2}{2} \omega_1=\frac{2 \mathrm{MR}^2}{3} \omega_2 \quad \ldots[\text { From (i)] } \\ \therefore \quad & \omega_2=\frac{3}{4} \omega_1=\frac{3}{4} \omega \quad \ldots\left(\because \omega_1=\omega\right)\end{aligned}$$

Question58

A solid cylinder of mass ' M ' and radius ' R ' rolls down an inclined plane of height ' h '. When it reaches the foot of the plane, its rotational kinetic energy is ($\mathrm{g}=$ acceleration due to gravity)

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Options:

A. $\frac{\mathrm{Mgh}}{3}$

B. $\frac{\mathrm{Mgh}}{6}$

C. $\frac{\mathrm{Mgh}}{4}$

D. $\frac{\mathrm{Mgh}}{2}$

Answer: A

Solution:

From the law of conservation of energy, we have

Potential energy = Translational kinetic energy + Rotational kinetic energy

or $\mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{I}\omega^2$

or $\mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}\mathrm{mr}^2\right)\omega^2 = \frac{3}{4}mr^2{\omega^2}^2$

or $\quad \omega^2 = \frac{4\mathrm{gh}}{3\mathrm{r}^2}$

Now the rotational kinetic energy $= \frac{1}{2}\mathrm{I}\omega^2$

$\therefore \quad$ Substituting for ω^2 and I, we have,

Rotational kinetic energy $= \frac{1}{2}\left(\frac{1}{2}\mathrm{mr}^2\right)\frac{4\mathrm{gh}}{3\mathrm{r}^2}$

$= \frac{\mathrm{Mgh}}{3} \quad \ldots(\because M = m)$

Question59

A disc and a ring both have same mass and radius. The ratio of moment of inertia of the disc about its diameter to that of a ring about a tangent in its plane is

Options:

A. 1 : 2

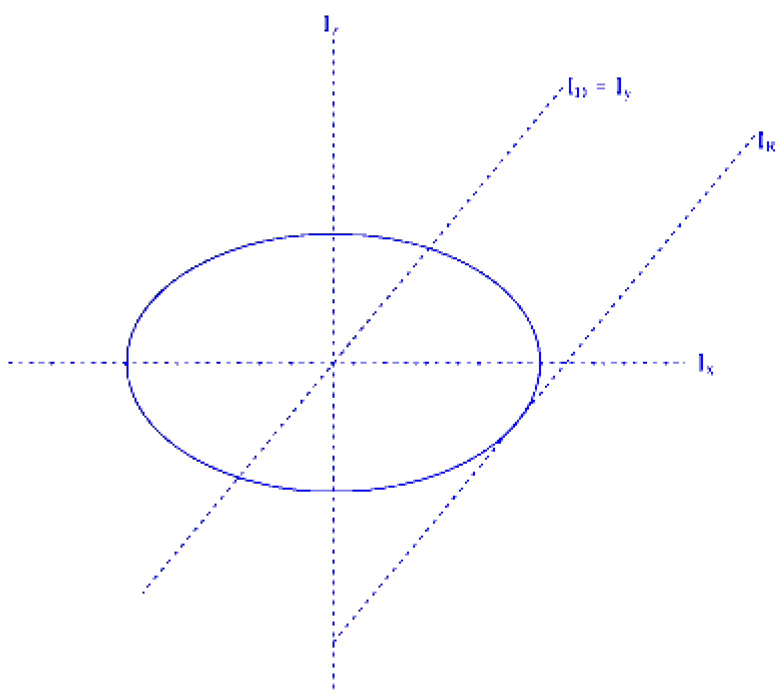
B. 1 : 4

C. 1 : 6

D. 1 : 8

Answer: C

Solution:



For ring, by using perpendicular and parallel axis theorem,

$$I_R = I_y + Mx^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$$\therefore \quad \frac{I_D}{I_R} = \frac{MR^2}{4} \times \frac{2}{3MR^2}$$

$$\therefore \quad \frac{I_D}{I_R} = \frac{1}{6}$$

Question60

A rotating body has angular momentum ' L '. If its frequency is doubled and kinetic energy is halved, its angular momentum will be

Options:

A. $\frac{L}{4}$

B. $\frac{L}{2}$

C. 2L

D. 4L

Answer: A

Solution:

Angular momentum of a particle performing UCM

$L = I\omega \quad .... (i)$

Kinetic energy, $k = \frac{1}{2}I\omega^2 \quad ... (ii)$

$\therefore \quad L = \frac{2k}{\omega} \quad ...$[From (i) and (ii)]

$$\therefore \quad \frac{L_1}{L_2} = \frac{k_1}{k_2} \times \frac{\omega_2}{\omega_1}$$

$$\frac{L}{L_2} = \frac{1}{\frac{1}{2}} \times \frac{2}{1} \quad ... \text{(given)}$$

$$\therefore \quad L_2 = \frac{L}{4}$$

Question61

A solid cylinder of mass M and radius R is rotating about its geometrical axis. A solid sphere of the same mass and same radius is also rotating about its diameter with an angular speed half that of the cylinder. The ratio of the kinetic energy of rotation of the sphere to that of the cylinder will be

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Options:

A. 1 : 4

B. 1 : 5

C. 2 : 3

D. 3 : 2

Answer: B

Solution:

$I_{\text{sphere}} = I_S = \frac{2}{5} mR^2$ and $I_{\text{cylinder}} = I_c = \frac{1}{2} mR^2$

Let $\omega_S =$ Angular speed of sphere,

$\omega_c =$ Angular speed of cylinder.

It is given that, $\omega_s = \frac{\omega_c}{2}$

$$\therefore \quad \frac{K \cdot E_{\text{sphere}}}{K \cdot E_{\text{cylinder}}} = \frac{I_s \omega_s^2}{I_c \omega_c^2} = \frac{\frac{2}{5} mR^2 \times \left(\frac{\omega_c}{2}\right)^2}{\frac{1}{2} mR^2 \times {\omega_c}^2}$$

$$\frac{K \cdot E_{\text{sphere}}}{K \cdot E_{\text{cylinder}}} = \frac{1}{5}$$

Question62

The earth is assumed to be a sphere of radius ' R ' and mass ' M ' having period of rotation ' T '. The angular momentum of earth about its axis of rotation is

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Options:

A. $\frac{2\pi MR^2}{5\ T}$

B. $\frac{4\pi MR^2}{5\ T}$

C. $\frac{\mathrm{MR}^2 \mathrm{~T}}{2 \pi}$

D. $\frac{\mathrm{MR}^2 \mathrm{~T}}{4 \pi}$

Answer: B

Solution:

The angular momentum of an object rotating about an axis is given by the formula:

$L=I \omega$

where L is the angular momentum, I is the moment of inertia of the object, and ω is the angular velocity.

For a solid sphere, the moment of inertia I about an axis through its center is:

$I=\frac{2}{5} M R^2$

where M is the mass and R is the radius of the sphere.

The angular velocity ω can be expressed in terms of the period of rotation T as:

$\omega=\frac{2 \pi}{T}$

Plugging these expressions into the angular momentum formula, we have:

$L=\left(\frac{2}{5} M R^2\right)\left(\frac{2 \pi}{T}\right)$

Simplifying, we get:

$L=\frac{4 \pi M R^2}{5 T}$

Therefore, the angular momentum of the Earth about its axis of rotation is given by:

Option B:

$\frac{4 \pi \mathrm{MR}^2}{5 \mathrm{~T}}$

Question63

Two loops ' A ' and ' B ' of radii ' R_1 ' and ' R_2 ' are made from uniform wire. If moment of inertia of ' A ' is ' I_{A} ' and that ' B ' is ' I_{B} ', then $\mathrm{R}_2 / \mathrm{R}_1$ is $\left[\frac{\mathrm{I}_{\mathrm{A}}}{\mathrm{I}_{\mathrm{B}}}=27\right]$

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Options:

A. 1 : 6

B. 1 : 4

C. 1 : 3

D. 1 : 2

Answer: C

Solution:

The moment of inertia of a loop made from uniform wire about its center is given by:

$I = mR^2$

where m is the mass of the wire, and R is the radius of the loop.

Assuming that the wire is uniform, both loops A and B will have the same linear mass density. This means the mass of a loop is proportional to its circumference:

$m = \lambda \cdot 2\pi R$

where λ is the linear mass density.

Therefore, the moments of inertia for loops A and B are:

$I_A = (\lambda \cdot 2\pi R_1)R_1^2 = \lambda \cdot 2\pi R_1^3$

$I_B = (\lambda \cdot 2\pi R_2)R_2^2 = \lambda \cdot 2\pi R_2^3$

Given that $\frac{I_A}{I_B} = 27$, we have:

$\frac{\lambda \cdot 2\pi R_1^3}{\lambda \cdot 2\pi R_2^3} = 27$

This simplifies to:

$\frac{R_1^3}{R_2^3} = 27$

Taking the cube root of both sides:

$\frac{R_1}{R_2} = 3$

Thus, the ratio R_2/R_1 is:

$\frac{R_2}{R_1} = \frac{1}{3}$

Hence, the correct option is:

Option C $1 : 3$

Question64

In case of rotational dynamics, which one of the following statements is correct?

[$\vec{\omega}$ = angular velocity, $\overrightarrow{\mathrm{v}}$ = linear velocity

$\overrightarrow{\mathbf{r}}$ = radius vector, $\vec{\alpha}$ = angular acceleration

$\overrightarrow{\mathrm{a}}$ = linear acceleration, $\overrightarrow{\mathrm{L}}$ = angular momentum

$\overrightarrow{\mathrm{p}}$ = linear momentum, $\bar{\tau}$ = torque,

$\overrightarrow{\mathrm{f}}$ = centripetal force]

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Options:

A. $\overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{r}}\times\vec{\omega},\overrightarrow{\boldsymbol{\alpha}}=\overrightarrow{\mathbf{r}}\times\vec{a},\vec{L}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{p}},\vec{\tau}=\overrightarrow{\mathrm{f}}\times\overrightarrow{\mathrm{r}}$

B. $\overrightarrow{\mathrm{v}}=\vec{\omega}\times\overrightarrow{\mathrm{r}},\vec{\alpha}=\overrightarrow{\mathrm{a}}\times\overrightarrow{\mathrm{r}},\overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{p}}\times\overrightarrow{\mathrm{r}},\vec{\tau}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{f}}$

C. $\overrightarrow{\mathrm{v}}=\vec{\omega}\times\overrightarrow{\mathrm{r}},\vec{\alpha}=\overrightarrow{\mathrm{a}}\times\overrightarrow{\mathrm{r}},\overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{p}},\vec{\tau}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{f}}$

D. $\overrightarrow{\mathrm{v}}=\vec{\omega}\times\overrightarrow{\mathrm{r}},\vec{\alpha}=\overrightarrow{\mathrm{a}}\times\overrightarrow{\mathrm{r}},\overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{p}}\cdot\overrightarrow{\mathrm{r}},\vec{\tau}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{f}}$

Answer: C

Solution:

The correct statement in rotational dynamics is:

Option C:

$$\overrightarrow{\mathrm{v}}=\vec{\omega}\times\overrightarrow{\mathrm{r}},\quad \vec{\alpha}=\overrightarrow{\mathrm{a}}\times\overrightarrow{\mathrm{r}},\quad \overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{p}},\quad \vec{\tau}=\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{f}}$$

Explanation:

Linear velocity $(\overrightarrow{\mathrm{v}})$:

The linear velocity of a point in rotational motion is given by the cross product of the angular velocity vector $(\vec{\omega})$ and the radius vector $(\overrightarrow{\mathrm{r}})$:

$\overrightarrow{\mathrm{v}}=\vec{\omega} \times \overrightarrow{\mathrm{r}}$

This shows the perpendicular relationship between linear velocity and both the radius vector and the angular velocity.

Angular acceleration $(\vec{\alpha})$:

The angular acceleration and its relation to linear acceleration can often be represented with:

$\vec{\alpha}=\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{r}}$

Though, if specified to mean tangential component in a circular path, it directly relates to the tangential acceleration.

Angular momentum $(\overrightarrow{\mathrm{L}})$:

The angular momentum of a particle with respect to a point is defined as:

$\overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{r}} \times \overrightarrow{\mathrm{p}}$

where $\overrightarrow{\mathrm{p}}$ is the linear momentum $(m v)$ of the particle.

Torque $(\vec{\tau})$:

Torque is defined as the cross product of the radius vector $(\overrightarrow{\mathrm{r}})$ and the force vector $(\overrightarrow{\mathrm{f}})$:

$\vec{\tau}=\overrightarrow{\mathrm{r}} \times \overrightarrow{\mathrm{f}}$

This defines the rotational effect of a force applied at a distance from a pivot.

Each component is consistent with the right-hand rule and the classical definitions in rotational dynamics.

Question65

Ratio of radius of gyration of a circular disc to that of circular ring each of same mass and radius around their respective axes is

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Options:

A. $\sqrt{2}: 1$

B. $\sqrt{2} : \sqrt{3}$

C. $\sqrt{3} : \sqrt{2}$

D. $1 : \sqrt{2}$

Answer: D

Solution:

The radius of gyration (k) is related to the moment of inertia (I) and the mass (M) of an object by the formula:

$$k = \sqrt{\frac{I}{M}}$$

For a circular disc:

Moment of inertia (I_{disc}) about its central axis is given by:

$$I_{\text{disc}} = \frac{1}{2} M R^2$$

Radius of gyration (k_{disc}) is:

$$k_{\text{disc}} = \sqrt{\frac{I_{\text{disc}}}{M}} = \sqrt{\frac{\frac{1}{2} M R^2}{M}} = \sqrt{\frac{1}{2}} R$$

For a circular ring:

Moment of inertia (I_{ring}) about its central axis is given by:

$$I_{\text{ring}} = M R^2$$

Radius of gyration (k_{ring}) is:

$$k_{\text{ring}} = \sqrt{\frac{I_{\text{ring}}}{M}} = \sqrt{\frac{M R^2}{M}} = R$$

Ratio of the radii of gyration:

The ratio $k_{\text{disc}} : k_{\text{ring}}$ is:

$$\sqrt{\frac{1}{2}} R : R = \frac{\sqrt{2}}{2} R : R = \frac{\sqrt{2}}{2} : 1 = \frac{1}{\sqrt{2}} : 1 = 1 : \sqrt{2}$$

Therefore, the ratio of the radius of gyration of a circular disc to that of a circular ring, each with the same mass and radius, is:

Option D: $1 : \sqrt{2}$

Question66

Two circular loops P and Q of radii ' r ' and ' nr ' are made respectively from a uniform wire. Moment of inertia of loop Q about its axis is four times that of loop P about its axis. The value of ' n ' is

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Options:

A. $(2)^{1/3}$

B. $(2)^{2/3}$

C. $(2)^{3/4}$

D. $(2)^{1/4}$

Answer: B

Solution:

The moment of inertia of a circular loop about its axis is given by the formula:

$I = mr^2$

where m is the mass of the loop and r is the radius.

Since the loops are made from the same uniform wire, the mass of the wire is proportional to its length. Therefore, the mass of each loop is proportional to its circumference:

$m_P \propto 2\pi r$

$m_Q \propto 2\pi(nr)$

The ratio of their masses is:

$\frac{m_Q}{m_P} = \frac{nr}{r} = n$

Thus, the mass of loop Q, $m_Q = n \cdot m_P$.

Using the moment of inertia formula, the moments of inertia for P and Q are:

$I_P = m_P r^2$

$I_Q = m_Q(nr)^2 = n \cdot m_P(nr)^2 = n \cdot m_P \cdot n^2 \cdot r^2 = n^3 m_P r^2$

Since it is given that $I_Q = 4I_P$:

$n^3 m_P r^2 = 4m_P r^2$

Cancelling $m_P r^2$ from both sides, we get:

$n^3 = 4$

Taking the cube root of both sides:

$n = \sqrt[3]{4} = 2^{2/3}$

Thus, the value of n is option B: $(2)^{2/3}$.

Question67

A solid sphere of mass ' m ', radius ' R ', having moment of inertia about an axis passing through center of mass as 'I' is recast into a disc of thickness ' t ' whose moment of inertia about an axis passing through the rim (edge) \& perpendicular to plane remains 'I'. Then the radius of disc is

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Options:

A. $\frac{2\mathrm{R}}{\sqrt{15}}$

B. $\left(\sqrt{\frac{2}{15}}\right)\mathrm{R}$

C. $\frac{4\mathrm{R}}{\sqrt{15}}$

D. $\frac{\mathrm{R}}{4}$

Answer: A

Solution:

The moment of inertia of a solid sphere

$= \frac{2}{5}\mathrm{MR}^2$

Moment of inertia of a disc through its rim,

$= \frac{1}{2}\mathrm{MR}^2 + \mathrm{MR}^2 = \frac{3}{2}\mathrm{MR}^2$

Since both the moment of inertias are equal, $\therefore \quad \frac{2}{5}\mathrm{MR}^2 = \frac{3}{2}\mathrm{Mr}^2$, where r is the radius of the disc

$\therefore \quad r=\frac{2R}{\sqrt{15}}$

Question68

An inclined plane makes an angle of 30° with the horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration ($g=$ acceleration due to gravity, $\sin 30^{\circ}=0.5$)

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Options:

A. $\frac{2g}{3}$

B. $\frac{5g}{14}$

C. $\frac{g}{3}$

D. $\frac{5g}{7}$

Answer: B

Solution:

$$a=\frac{g \sin \theta}{\left(1+\frac{K^2}{R^2}\right)}=\frac{g \sin 30^{\circ}}{\left(1+\frac{2}{5}\right)}$$

$$\therefore \quad a=\frac{5g}{7} \times 0.5=\frac{5g}{14}$$

Question69

An annular ring has mass 10 kg and inner and outer radii are 10 m and 5 m respectively. Its moment of inertia about an axis passing through its centre and perpendicular to its plane is

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Options:

A. 525 kgm^2

B. 625 kgm^2

C. 525 gcm^2

D. 625 gcm^2

Answer: B

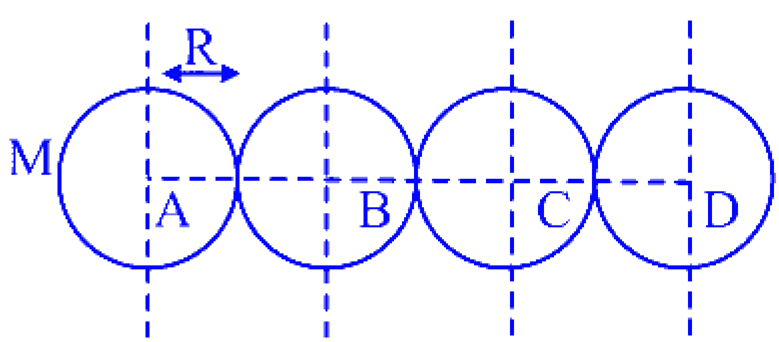
Solution:

The moment of inertia of the annular ring is

$$I=\frac{1}{2} M\left(R_1^2+R_2^2\right)$$
$$=\frac{10(100+25)}{2}=625 \mathrm{~kgm}^2$$

Question70

Four identical uniform solid spheres each of same mass 'M' and radius 'R' are placed touching each other as shown in figure, with centres A, B, C, D. $\mathrm{I_A}, \mathrm{I_B}, \mathrm{I_C}$ and $\mathrm{I_D}$ are the moment of inertia of these spheres respectively about an axis passing through centre and perpendicular to the plane. The difference in $\mathrm{I_A}$, and $\mathrm{I_B}$ is



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Options:

A. 24 MR^2

B. 32 MR^2

C. 56 MR^2

D. 80 MR^2

Answer: B

Solution:

Using the parallel axes theorem, the M.I. of the system about the axis passing through the centre of the sphere A is

$$I_A = I_A' + I_B' + I_{C'} + I_D'$$
$$I_A = \frac{2}{5}MR^2 + \left(\frac{2}{5}MR^2 + 4MR^2\right) + \left(\frac{2}{5}MR^2 + 16MR^2\right) + \left(\frac{2}{5}M^2 + 36MR^2\right)$$

$$\therefore \quad I_A = 57.6MR^2$$

The M.I. about sphere B is,

$$I_B = \frac{2}{5}MR^2 + \left(\frac{2}{5}MR^2 + 4MR^2\right) + \left(\frac{2}{5}MR^2 + 16MR^2\right) + \left(\frac{2}{5}MR^2 + 4MR^2\right)$$

$$I_B = 25.6MR^2$$
$$\therefore \quad I_A - I_B = 57.6MR^2 - 25.6MR^2 = 32MR^2$$

Question71

A solid cylinder and a solid sphere having same mass and same radius roll down on the same inclined plane. The ratio of the acceleration of the cylinder 'a_c' to that of sphere 'a_s' is

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Options:

A. $\frac{11}{15}$

B. $\frac{13}{14}$

C. $\frac{15}{14}$

D. $\frac{14}{15}$

Answer: D

Solution:

For sphere: M.I. $I_S = \frac{2}{5}MR^2$

For cylinder: M.I. $I_C = \frac{1}{2}MR^2$

Acceleration: $a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$

$$\therefore \quad \frac{a_c}{a_s} = \frac{1+\frac{I_s}{MR^2}}{1+\frac{I_C}{MR^2}} = \frac{MR^2+I_s}{MR^2+I_c} = \frac{MR^2+\frac{2}{5}MR^2}{MR^2+\frac{1}{2}MR^2}$$

$$\therefore \quad \frac{a_c}{a_s} = \frac{7}{5}\times\frac{2}{3} = \frac{14}{15}$$

Question72

A mass 'M' is moving with constant velocity parallel to X-axis. Its angular momentum with respect to the origin is

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Options:

A. constant

B. zero

C. decreasing

D. increasing

Answer: A

Solution:

The angular momentum $(\vec{L})$ of a particle with respect to a point (in this case, the origin) is given by the cross product of the position vector $(\vec{r})$ and the linear momentum of the particle $(\vec{p})$, which is the product of the mass M and its velocity $\vec{v}$:

$\vec{L} = \vec{r} \times \vec{p}$

If a mass M is moving with constant velocity parallel to the X-axis, then its momentum $\vec{p}$ is also constant both in magnitude and direction, given by:

$\vec{p} = M\vec{v}$

where $\vec{v}$ is the constant velocity vector of the mass parallel to the X-axis.

The position vector $\vec{r}$ is a vector from the origin to the location of the mass M. Since the mass is moving with a constant velocity and not approaching or moving away from the origin, the perpendicular distance from the origin to the line of motion (essentially, the "arm" of the moment arm) remains constant.

The cross product of two vectors yields a vector that is perpendicular to the plane formed by the two original vectors and its magnitude is given by:

$|\vec{L}| = |\vec{r}||\vec{p}| \sin\theta$

where θ is the angle between $\vec{r}$ and $\vec{p}$.

Since the mass is moving parallel to the X-axis, the angle θ between the position vector from the origin and the momentum vector is constant, and so is the sine of that angle. Thus, the product $|\vec{r}||\vec{p}|$ is also constant.

Therefore, the magnitude of the angular momentum $|\vec{L}|$ is constant, and because the mass M is not changing its direction of motion or speed, the direction of the angular momentum vector is also constant.

The correct answer to the question is:

Option A: constant

since the angular momentum does not change with time when the velocity is constant and the direction of motion is a straight line, and because there are no external forces or torques acting on the mass to change its state of motion or angular momentum.

Question73

A thin uniform rod of mass 'm' and length 'P' is suspended from one end which can oscillate in a vertical plane about the point of intersection. It is pulled to one side and then released. It passes through the equilibrium position with angular speed 'ω'. The kinetic energy while passing through mean position is

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Options:

A. $\mathrm{ml}^2\omega^2$

B. $\frac{\mathrm{m}l^2\omega^2}{4}$

C. $\frac{\mathrm{m}l^2\omega^2}{6}$

D. $\frac{\mathrm{ml}^2\omega^2}{12}$

Answer: C

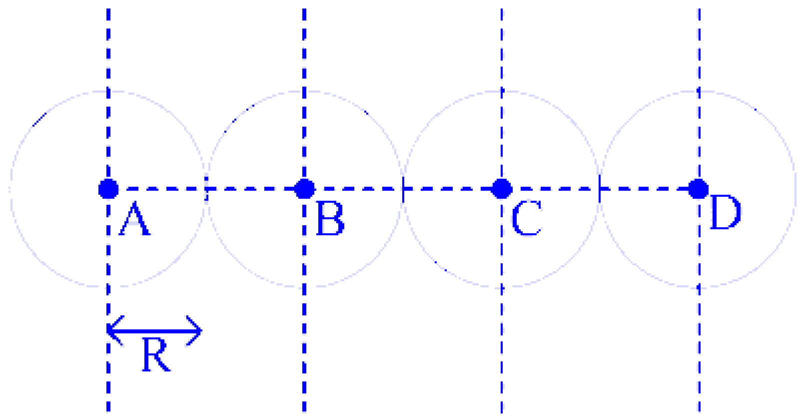
Solution:

The kinetic energy of the rod while passing through the mean position will be,

$$\begin{aligned} \text{K.E.} &= \frac{1}{2}\mathrm{I}\omega^2 \\ &= \frac{1}{2}\frac{\mathrm{m}l^2}{3} \times \omega^2 = \frac{\mathrm{m}l^2\omega^2}{6} \end{aligned}$$

Question74

Four identical uniform solid spheres each of same mass M and radius R are placed touching each other as shown in figure with centres A, B, C, D. I_A, I_B, I_C, I_D are the moment of inertia of these spheres respectively about an axis passing through centre and perpendicular to the plane, then



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Options:

A. $I_A > I_B > I_C > I_D$

B. $I_D > I_C > I_B > I_A$

C. $I_A = I_D > I_B = I_C$

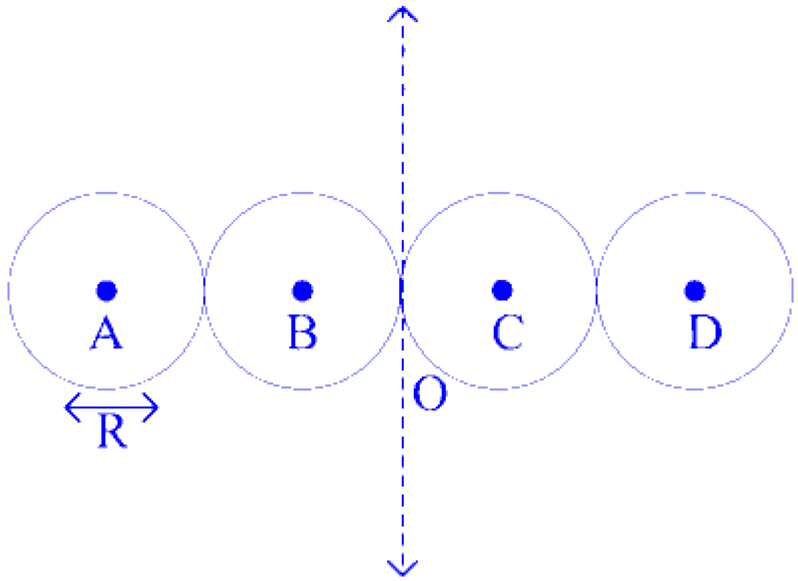
D. $I_A = I_D < I_B = I_C$

Answer: C

Solution:

The four identical uniform solid spheres are shown below. The radius of each sphere is R.

Lets take centre of the system at O.



Then, distance from the centre for $r_B = r_C$ and $r_A = r_D$

$\therefore$ Moment of inertial, $M_0 I = Mr^2$, for each spheres will be I_A, I_B, I_C and I_D.

$\therefore$ $I_A = I_D$ and $I_B = I_C$

Since, $r_B = r_C < r_A = r_D$

Therefore, $I_A = I_D > I_B = I_C$

Question75

A thin uniform rod AB of mass m and length l is hinged at one end A to the ground level. Initially the rod stands vertically and is allowed to fall freely to the ground in the vertical plane. The angular velocity of the rod when its end B strikes the ground is ($g =$ acceleration due to gravity)

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Options:

A. $\sqrt{\frac{g}{l}}$

B. $\sqrt{\frac{mg}{l}}$

C. $\sqrt{\frac{3g}{l}}$

D. $\sqrt{\frac{mg}{3l}}$

Answer: C

Solution:

Let the length of the rod AB be l and mass of the rod be m.

Moment of inertia of the uniform rod about an axis passing through one of its end V given as,

$\mathrm{MI} = \frac{ml^2}{3}$

Rotational kinetic energy, $(\mathrm{KE})_{\mathrm{rod}} = \frac{1}{2} l\omega^2$

Initially the rod is held, vertical, so potential energy of rod is given by $(\mathrm{PE})_i = mg\frac{I}{2} \quad (\because \frac{l}{2} = \mathrm{COM})$

Initial kinetic energy, $(\mathrm{KE})_i = 0$

The rod is allowed to fall, when it just touches the ground its potential energy becomes zero, i.e., $(\mathrm{PE})_f = 0$

Let, angular velocity of rod at that instant be ω, then kinetic energy at that instant is

$(\mathrm{KE})_f = \frac{1}{2} /\omega^2$

From conservation of energy,

$(\mathrm{KE})_i + (\mathrm{PE})_i = (\mathrm{KE})_f + (\mathrm{PE})_f$

$$\Rightarrow \quad 0 + mg\frac{l}{2} = \frac{1}{2}l\omega^2 + 0$$
$$\Rightarrow \quad mgl = l\omega^2$$
$$\Rightarrow \quad mgl = \frac{ml^2}{3}\omega^2$$
$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{l}}$$

Question76

A rigid body rotates with an angular momentum L. If its rotational kinetic energy is made four times, its angular momentum will become

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Options:

A. 4 L

B. 16 L

C. $\sqrt{2}$ L

D. 2 L

Answer: D

Solution:

Angular momentum,

$$\begin{aligned} &\mathrm{L} = \sqrt{2\mathrm{KI}} \\ &\mathrm{K} = 4\,\mathrm{K} \quad \text{(given)} \\ \therefore \quad &\mathrm{L} = \sqrt{2 \times 4\mathrm{KI}} = 2\sqrt{2\mathrm{KI}} = 2\,\mathrm{L} \end{aligned}$$

Question77

The rotational kinetic energy and translational kinetic energy of a rolling body are same, the body is

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Options:

A. disc

B. sphere

C. cylinder

D. ring

Answer: D

Solution:

Translational kinetic energy of a ring is:

$KE_{Trans} \quad = \frac{1}{2} mv^2$

Rotational kinetic energy of the ring is:

$$\begin{aligned} KE_{Rolling} & = \frac{1}{2} I^2 \\ & = \frac{1}{2} mR^2 \omega^2 \\ KE_{Rolling} & = \frac{1}{2} mv^2 \quad (\because v = \omega R) \end{aligned}$$

Question78

A solid cylinder of mass $3\ kg$ is rolling on a horizontal surface with velocity $4\ m/s$. It collides with a horizontal spring whose one end is fixed to rigid support. The force constant of material of spring is $200\ N/m$. The maximum compression produced in the spring will be (assume collision between cylinder & spring be elastic)

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Options:

A. 0.7 m

B. 0.2 m

C. 0.5 m

D. 0.6 m

Answer: D

Solution:

At maximum compression, the solid cylinder will stop.

So loss in K.E. of cylinder = Gain in P.E. of spring

$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2$

$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{1}{2}kx^2$

$\therefore \quad \frac{3}{4}mv^2 = \frac{1}{2}kx^2$

$\therefore \quad \frac{3}{4} \times 3 \times (4)^2 = \frac{1}{2} \times 200 \times x^2$

$\therefore \quad \frac{36}{100} = x^2 \Rightarrow x = 0.6 \text{ m}$

Question79

A thin wire of length 'L' and uniform linear mass density 'm' is bent into a circular coil. The moment of inertia of this coil about tangential axis and in plane of the coil is

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Options:

A. $\frac{3\text{ mL}^2}{5\pi^2}$

B. $\frac{3\text{ mL}^3}{8\pi^2}$

C. $\frac{3\text{ mL}^3}{4\pi^2}$

D. $\frac{3\text{ mL}^2}{7\pi^2}$

Answer: B

Solution:

$\therefore$ Moment of inertia of thin wire:

$$I = \frac{M^2}{2}$$
$$M = v \times m \text{ and } L = 2\pi R$$
$$R = \frac{L}{2\pi}$$
$$\therefore \quad I = \frac{Lm}{2}\left(\frac{L}{2\pi}\right)^2$$
$$I = \frac{mL^3}{8\pi^2}$$

$\therefore$ Using Parallel axis theorem:

$$I' = I + MR^2$$
$$I = \frac{mL^3}{8\pi^2} + Lm\left(\frac{L}{2\pi}\right)^2$$
$$I = \frac{3mL^3}{8\pi^2}$$

Question80

In P^{th} second, a particle describes angular displacement of 'β' rad. If it starts from rest, the angular acceleration is

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Options:

A. $\frac{\beta}{P}$

B. $\frac{\beta}{(P-1)}$

C. $\frac{2\beta}{(2P-1)}$

D. $\frac{(2\beta+1)}{(2P-1)}$

Answer: C

Solution:

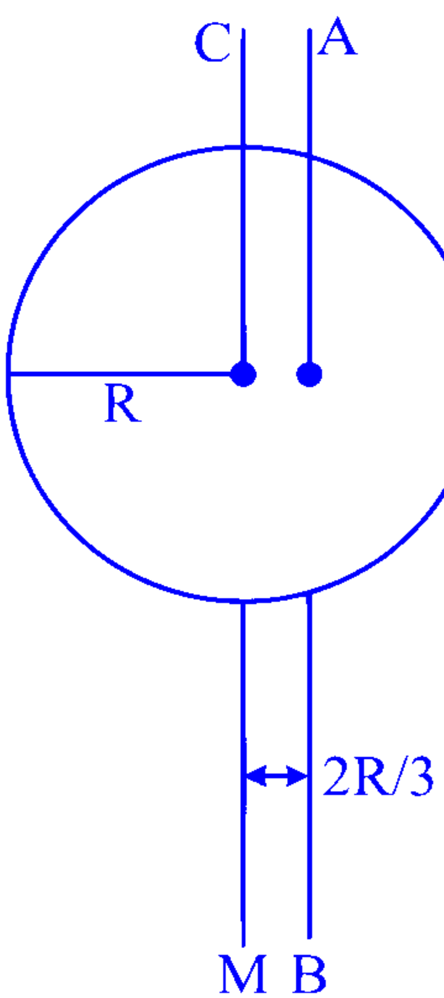
The equation for the angular displacement of the particle that starts from rest is given as:

$\beta = 0 + \frac{1}{2}\alpha(2P - 1)$

$\therefore$ The angular acceleration of the particle is $\alpha = \frac{2\beta}{(2P-1)}$

Question81

I_1 is the moment of inertia of a circular disc about an axis passing through its centre and perpendicular to the plane of disc. I_2 is its moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2R}{3}$ from centre. The ratio of I_1 and I_2 is $x : 17$. The value of 'x' is (R = radius of the disc)



Options:

A. 9

B. 12

C. 15

D. 17

Answer: A

Solution:

Using Parallel axis theorem, $\mathrm{I}_2=\mathrm{I}_1+\mathrm{Mh}^2$

For a disc, $I_1=\frac{1}{2} M R^2$ and given that, $h=\frac{2 R}{3}$

$$\therefore \quad \mathrm{I}_2=\frac{1}{2} \mathrm{MR}^2+\mathrm{M}\left(\frac{4 \mathrm{R}^2}{9}\right)$$

$$\therefore \quad \mathrm{I}_2=\frac{1}{2} \mathrm{MR}^2\left(1+\frac{8}{9}\right)=\mathrm{I}_1 \times \frac{17}{9}$$

$$\therefore \quad \frac{\mathrm{I}_1}{\mathrm{I}_2}=\frac{9}{17}$$

Question82

A thin uniform circular disc of mass 'M' and radius 'R' is rotating with angular velocity 'ω', in a horizontal plane about an axis passing through its centre and perpendicular to its plane. Another disc of same radius but of mass $\left(\frac{M}{2}\right)$ is placed gently on the first disc co-

axially. The new angular velocity will be

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Options:

A. $\frac{2}{3}\omega$

B. $\frac{4}{5}\omega$

C. $\frac{5}{4}\omega$

D. $\frac{3}{2}\omega$

Answer: A

Solution:

Angular momentum $= \mathrm{I}\omega$

By conservation of angular momentum, $\mathrm{I}_1\omega_1 = \mathrm{I}_2\omega_2$

Here, $I_1 = \frac{MR^2}{2}, I_2 = \frac{(M+M/2)}{2}R^2 = \frac{3MR^2}{4}$

$$\therefore \quad \frac{\mathrm{MR}^2}{2}\omega_1 = \frac{3\mathrm{MR}^2}{4}\omega_2$$

$$\therefore \quad \omega_2 = \frac{2}{3}\omega_1$$

Question83

Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

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Options:

A. 1 : 2

B. $\sqrt{2}$: 1

C. 2 : 1

D. 1 : $\sqrt{2}$

Answer: D

Solution:

The equation for angular momentum is

$L = \sqrt{2\ K_{Rot} \quad \times I}$

So, $L \propto \sqrt{I}$

$\therefore$ The ratio of angular momentum of the two bodies is

$\frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$

$\frac{L_1}{L_2} = \sqrt{\frac{I}{2I}} \quad \ldots.(\text{given } I_2 = 2I)$

$\therefore \quad \frac{L_1}{L_2} = \frac{1}{\sqrt{2}}$

Question84

From a disc of mass 'M' and radius 'R', a circular hole of diameter 'R' is cut whose rim passes through the centre. The moment of inertia of the remaining part of the disc about perpendicular axis passing through the centre is

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Options:

A. $\frac{13MR^2}{32}$

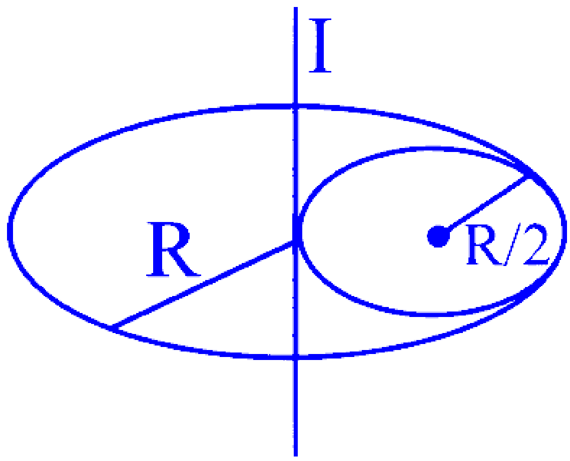
B. $\frac{11MR^2}{32}$

C. $\frac{9MR^2}{32}$

D. $\frac{7MR^2}{32}$

Answer: A

Solution:



Moment of inertia of disc is given by

$I_{disc} = I_r + I_{hole} \quad \ldots.\{I_r = \text{M.I. of remaining part}\}$

$\therefore \quad I_r = I_{disc} - I_{hole} \quad \ldots..\text{(i)}$

$I_{disc} = \frac{MR^2}{2} \ldots..\text{(ii)}$

By parallel axes theorem we get,

$$I_{hole} = \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$$

$$\ldots.\left\{\begin{array}{l}\because M_{hole} = \frac{M_{disc}}{4} \\ \because \text{ the surface density is same}\end{array}\right\}$$

$\therefore \quad I_{hole} = \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right] \ldots..\text{(iii)}$

Substituting eq (iii) and eq (ii) in eq (i) we get,

$$
\begin{aligned}
I_r &= \frac{MR^2}{2} - \frac{MR^2}{32} - \frac{MR^2}{16} \\
&= MR^2\left[\frac{1}{2} - \frac{1}{32} - \frac{1}{16}\right] \\
&= \frac{13}{32} MR^2
\end{aligned}
$$

Question85

A particle moves along a circular path with decreasing speed. Hence

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Options:

A. its resultant acceleration is towards the centre.

B. it moves in a spiral path with decreasing radius.

C. the direction of angular momentum remains constant.

D. its angular momentum remains constant

Answer: C

Solution:

The direction of angular moment remains constant as the angular momentum is a vector quantity, and its direction is perpendicular to the plane of motion. As the speed decreases, the linear momentum decreases, but the angular momentum remains constant due to the conservation of angular momentum.

Question86

Radius of gyration of a thin uniform circular disc about the axis passing through its centre and perpendicular to its plane is K_c.

Radius of gyration of the same disc about a diameter of the disc is K_d. The ratio $K_c : K_d$ is

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Options:

A. $\sqrt{2} : 1$

B. $1 : \sqrt{2}$

C. $2 : 1$

D. $1 : 4$

Answer: A

Solution:

Let the radius of the disc be R

$$\therefore \quad \mathrm{K_c} = \frac{\mathrm{R}}{\sqrt{2}}$$
$$\therefore \quad \mathrm{K_d} = \frac{\mathrm{R}}{2}$$

Taking the ratio,

$$\therefore \quad \frac{\mathrm{K_c}}{\mathrm{K_d}} = \frac{\frac{\mathrm{R}}{\sqrt{2}}}{\frac{\mathrm{R}}{2}} = \frac{\sqrt{2}}{1}$$

Question87

A disc has mass M and radius R. How much tangential force should be applied to the rim of the disc, so as to rotate with angular velocity 'ω' in time t ?

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Options:

A. $\frac{MR\omega}{4t}$

B. $\frac{\mathrm{MR}\omega}{2\mathrm{t}}$

C. $\frac{\mathrm{MR}\omega}{\mathrm{t}}$

D. $\mathrm{MR}\omega\mathrm{t}$

Answer: B

Solution:

Torque: $\tau=\mathrm{I}\alpha=\frac{\mathrm{MR}^2}{2}\times\frac{\omega}{\mathrm{t}}$

$\therefore \quad \tau=\frac{\mathrm{MR}^2\omega}{2\mathrm{t}}$

But $\tau=\mathrm{R}\times\mathrm{F}$

$\therefore \quad \mathrm{F}=\frac{\tau}{\mathrm{R}}=\frac{\mathrm{MR}\omega}{2\mathrm{t}}$

Question88

What is the moment of inertia of the electron moving in second Bohr orbit of hydrogen atom? [$\mathrm{h}=$ Planck's constant, $\mathrm{m}=$ mass of electron, $\varepsilon_0=$ permittivity of free space, $\mathrm{e}=$ charge on electron]

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Options:

A. $\frac{4\varepsilon_0^2h^4}{\pi^2me^4}$

B. $\frac{8m\varepsilon_0^2h^4}{\pi^2e^4}$

C. $\frac{16\varepsilon_0^2h^4}{\pi^2me^4}$

D. $\frac{\varepsilon_0^2 h^4}{16\pi^2 \mathrm{me}^4}$

Answer: C

Solution:

Moment of Inertia $\mathrm{I}=\mathrm{MR}^2$

Radius of the n^{th} Bohr orbit is,

$r_n=\frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$

For $\mathrm{n}=2$,

$r_2=\frac{4\varepsilon_0 h^2}{\pi m e^2}$

$\therefore$ Moment of inertia of the electron in the 2^{nd} orbit is

$$\begin{aligned}\text{M.I} &= m \times\left[\frac{4 \varepsilon_0 h^2}{\pi m e^2}\right]^2 \\ &= m \times \frac{16 \varepsilon_0^2 h^4}{\pi^2 m^2 e^4} \\ &= \frac{16 \varepsilon_0^2 h^4}{\pi^2 m e^4}\end{aligned}$$

Question89

Two spheres each of mass 'M' and radius $\frac{R}{2}$ are connected at the ends of massless rod of length '$2R$'. What will be the moment of inertia of the system about an axis passing through centre of one of the spheres and perpendicular to the rod?

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Options:

A. $\frac{2}{3}\mathrm{MR}^2$

B. $\frac{5}{2}\mathrm{MR}^2$

C. $\frac{5}{21} \mathrm{MR}^2$

D. $\frac{21}{5} \mathrm{MR}^2$

Answer: D

Solution:

From parallel axis theorem,

$I_o = I_c + Mh^2$

Let the moment of inertia of sphere 1 be

$I_1 = \frac{2}{5} M\left(\frac{R}{2}\right)^2 + M(2R)^2$

and,

Let the moment of inertia of sphere 2 be

$I_2 = \frac{2}{5} M\left(\frac{R}{2}\right)^2$

Moment of inertia of the rod $I_3 = 0$

$\therefore$ Moment of inertia of the system,

$$I = I_1 + I_2 + I_3$$
$$I = \frac{2}{5} M\left(\frac{R}{2}\right)^2 + M(2R)^2 + \frac{2}{5} M\left(\frac{R}{2}\right)^2$$
$$= \frac{4}{5} M\left(\frac{R}{2}\right)^2 + 4MR^2$$
$$= \frac{1}{5} MR^2 + 4MR^2$$
$$= \frac{21}{5} MR^2$$

Question90

The moment of inertia of a uniform square plate about an axis perpendicular to its plane and passing through the centre is $\frac{Ma^2}{6}$, where 'M' is the mass and 'a' is the side of square plate. Moment of

inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

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Options:

A. $\frac{Ma^2}{6}$

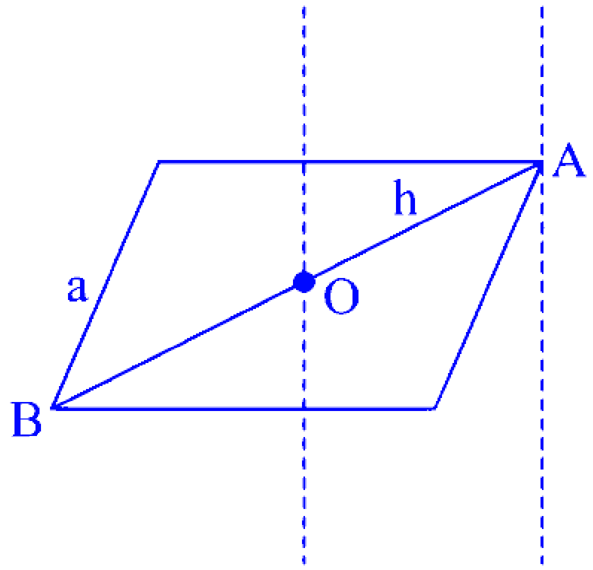
B. $\frac{2Ma^2}{3}$

C. $\frac{Ma^2}{3}$

D. $\frac{2Ma^2}{5}$

Answer: B

Solution:



$$I_O = \frac{Ma^2}{6}$$

$$AB = \sqrt{2a^2} = \sqrt{2}a$$

$$\therefore \quad AO = \frac{a}{\sqrt{2}} = h$$

$$\therefore \quad I_A = I_0 + Mh^2$$

$$= \frac{Ma^2}{6} + \frac{Ma^2}{2}$$
$$= \frac{8Ma^2}{12} = \frac{2}{3}Ma^2$$

Question91

If 'I' is moment of inertia of a thin circular disc about an axis passing through the tangent of the disc and in the plane of disc. The moment of inertia of same circular disc about an axis perpendicular to plane and passing through its centre is

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Options:

A. $\frac{4I}{5}$

B. $\frac{2I}{5}$

C. $\frac{4I}{3}$

D. $\frac{2I}{3}$

Answer: B

Solution:

M.I. of thin circular disc through the tangent in the plane of the disc is $\mathrm{I} = \frac{5}{4}\mathrm{MR}^2$

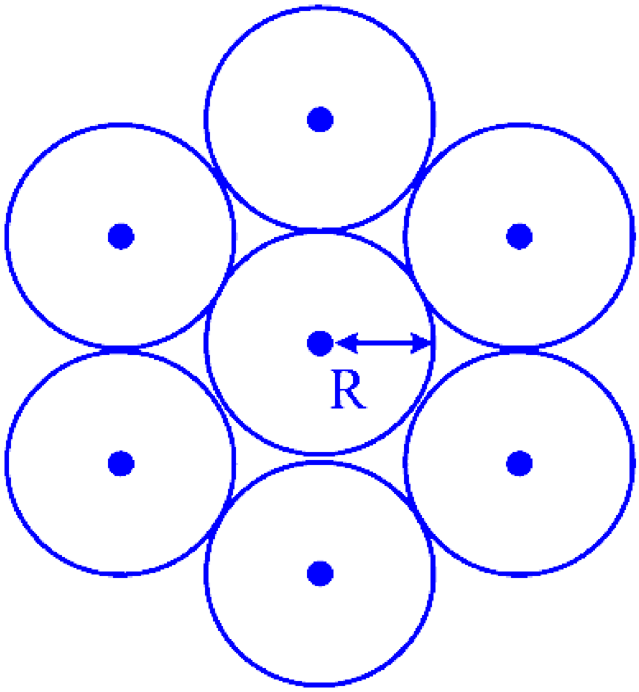
$\Rightarrow \mathrm{MR}^2 = \frac{4}{5}\mathrm{I}$

$\therefore$ M.I. of thin circular disc about an axis perpendicular to plane and passing through its

centre $= \frac{\mathrm{MR}^2}{2} = \frac{\left(\frac{4}{5}\mathrm{I}\right)}{2} = \frac{2\mathrm{I}}{5}$

Question92

Seven identical discs each of mass M and radius R are arranged in a hexagonal plane pattern so as to touch each neighbour disc as shown in the figure. The moment of inertia of the system of seven discs about an axis passing through the centre of central disc and normal to the plane of all discs is



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Options:

A. $\frac{7}{2}\ \mathrm{MR}^2$

B. $\frac{13}{2}\ \mathrm{MR}^2$

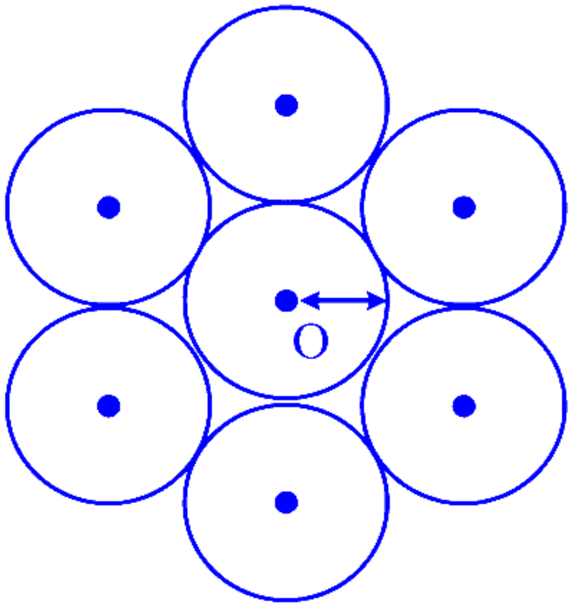
C. $\frac{29}{2}\ \mathrm{MR}^2$

D. $\frac{55}{2}\ \mathrm{MR}^2$

Answer: D

Solution:

M.I of a circular disc $= \frac{MR^2}{2}$



Using parallel axis theorem, M.I. about origin $I = I_{cm} + 6I$

where, I_{cm} = M. I of the central disc

I' = M. I of the each disc about the given axis.

$$\begin{aligned}
\therefore \quad I &= \frac{MR^2}{2} + 6\left(I_{cm} + MD^2\right) \\
&= \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + 4MR^2\right) \quad \ldots(\because D = 2R) \\
&= \frac{MR^2}{2} + 6\left(\frac{MR^2 + 8MR^2}{2}\right) \\
&= \frac{55MR^2}{2}
\end{aligned}$$

Question93

A particle of mass 'm' is rotating along a circular path of radius 'r' having angular momentum 'L'. The centripetal force acting on the particle is given by

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Options:

A. $\frac{L^2}{mr}$

B. $\frac{L^2}{mr^2}$

C. $\frac{mL^2}{r}$

D. $\frac{L^2}{mr^3}$

Answer: D

Solution:

$$\text{Centripetal force } = m^2 r$$
$$\therefore \quad \omega = \frac{L}{I} = \frac{L}{mr^2} \quad \ldots . (\because L = I\omega)$$
$$\therefore \quad F = m\left(\frac{L}{mr^2}\right)^2 r = \frac{mL^2 r}{m^2 r^4} = \frac{L^2}{mr^3}$$

Question94

A disc of radius R and thickness $\frac{R}{6}$ has moment of inertia I about an axis passing through its centre and perpendicular to its plane. Dise is melted and recast into a solid sphere. The moment of inertia of a sphere about its diameter is

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Options:

A. $\frac{I}{5}$

B. $\frac{I}{6}$

C. $\frac{I}{32}$

D. $\frac{I}{64}$

Answer: A

Solution:

M.I. of disc, $I = \frac{1}{2}\ \mathrm{MR}_{\mathrm{d}}^2$ ..... (i)

M.I. of sphere, $\mathrm{I}_{\text{sphere}} \quad = \frac{2}{5}\ \mathrm{MR}_{\mathrm{s}}^2$ .... (ii)

$\because$ volume of disc = volume of sphere

$\therefore \quad \pi \mathrm{R}_{\mathrm{d}}^2\left(\frac{\mathrm{R}_{\mathrm{d}}}{6}\right)=\frac{4}{3} \pi \mathrm{R}_{\mathrm{s}}^3$

$\therefore \quad \mathrm{R}_{\mathrm{d}}^3=8 \mathrm{R}_{\mathrm{s}}^3$

$\therefore \quad \mathrm{R}_{\mathrm{S}}=\frac{\mathrm{R}_{\mathrm{d}}}{2}$ .... (iii)

Substitute equation (iii) in equation (ii)

$$\therefore \quad I_{\text{sphere}} = \frac{2}{5} \mathrm{M}\left(\frac{\mathrm{R}_{\mathrm{d}}}{2}\right)^2=\frac{2}{5} \times \frac{1}{4} \mathrm{MR}_{\mathrm{d}}^2$$

$$= \frac{1}{5}\left(\frac{1}{2} \mathrm{MR}_{\mathrm{d}}^2\right)=\frac{1}{5} \quad \text{.... [from (i)]}$$

Question95

A square lamina of side 'b' has same mass as a disc of radius 'R' the moment of inertia of the two objects about an axis perpendicular to the plane and passing through the centre is equal. The ratio $\frac{b}{R}$ is

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Options:

A. $1 : 1$

B. $\sqrt{3} : 1$

C. $\sqrt{6} : 1$

D. $1 : \sqrt{3}$

Answer: B

Solution:

$$I_{\text{lamina}} = \frac{Mb^2}{6}$$

$$I_{\text{disc}} = \frac{MR^2}{2}$$

$$\text{Given } \frac{Mb^2}{6} = \frac{MR^2}{2}$$

$$\frac{b^2}{R^2} = 3$$

$$\therefore \quad \frac{b}{R} = \frac{\sqrt{3}}{1}$$

Question96

A solid sphere rolls without slipping on an inclined plane at an angle θ. The ratio of total kinetic energy to its rotational kinetic energy is

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Options:

A. $\frac{7}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{3}$

D. $\frac{5}{4}$

Answer: A

Solution:

Moment of Inertia of a solid sphere, $I = \frac{2}{5}MR^2$

Since there is no slipping,

$v = R\omega$

$\therefore$ Rotational kinetic energy

$$E_{rot} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5} \times M \times R^2 \times \omega^2$$

$$= \frac{MR^2\omega^2}{5}$$

$$= \frac{MV^2}{5} \quad .... \text{(i)}$$

Total kinetic energy

$$E_K = \frac{1}{2} I\omega^2 + \frac{1}{2} MV^2$$

$$= \frac{MV^2}{5} + \frac{MV^2}{2}$$

$$= \frac{7MV^2}{10} \quad ..... \text{(ii)}$$

Dividing (ii) by (i), we get,

$$\frac{E_K}{E_{rot}} = \frac{\left(\frac{7MV^2}{10}\right)}{\left(\frac{MV^2}{5}\right)} = \frac{7}{2}$$

Question97

Two discs of same mass and same thickness (t) are made from two different materials of densities 'd_1' and 'd_2' respectively. The ratio of the moment of inertia I_1 to I_2 of two discs about an axis passing through the centre and perpendicular to the plane of disc is

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Options:

A. $d_1 : d_2$

B. $d_2 : d_1$

C. $1 : d_1 d_2$

D. $1 : d_1^2 d_2$

Answer: B

Solution:

The ratio of moments of inertia of two discs of the same mass and same thickness but of different densities is given by $\frac{I_1}{I_2}=\frac{R_1^2}{R_2^2}=\frac{d_2}{d_1}$

Question98

The moment of inertia of a thin uniform rod of mass 'M' and length 'L' about an axis passing through a point at a distance $\frac{L}{4}$ from one of its ends and perpendicular to the length of the rod is

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Options:

A. $\frac{\mathrm{ML}^2}{48}$

B. $\frac{7\mathrm{ML}^2}{48}$

C. $\frac{5\mathrm{ML}^2}{48}$

D. $\frac{9\mathrm{ML}^2}{48}$

Answer: B

Solution:

The moment of inertia of the rod about an axis passing through the centre and perpendicular to its length is given by

$\mathrm{I}_0=\frac{\mathrm{ML}^2}{12}$

A point at a distance $\frac{\mathrm{L}}{4}$ from its end will also be at a distance $\frac{\mathrm{L}}{4}$ from the centre.

Hence by parallel axis theorem,

$\mathrm{I}=\mathrm{I}_0+\mathrm{M}\left(\frac{\mathrm{L}}{4}\right)^2=\frac{\mathrm{ML}^2}{12}+\frac{\mathrm{ML}^2}{16}=\frac{7\mathrm{ML}^2}{48}$

Question99

Two identical particles each of mass 'm' are separated by a distance 'd'. The axis of rotation passes through the midpoint of 'd' and is perpendicular to the length d. If 'K' is the average rotational kinetic energy of the system, then the angular frequency is

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Options:

A. $2 \mathrm{~d} \sqrt{\frac{\mathrm{m}}{\mathrm{K}}}$

B. $\frac{\mathrm{d}}{2} \sqrt{\frac{\mathrm{K}}{\mathrm{m}}}$

C. $\frac{2}{\mathrm{~d}} \sqrt{\frac{\mathrm{K}}{\mathrm{m}}}$

D. $\frac{d}{4} \sqrt{\frac{\mathrm{m}}{\mathrm{K}}}$

Answer: C

Solution:

Moment of inertia $\mathrm{I}=2 \mathrm{~m}\left(\frac{\mathrm{d}}{2}\right)^2=\frac{\mathrm{md}^2}{2}$

Kinetic energy $\mathrm{k}=\frac{1}{2} \mathrm{I} \omega^2$

$$\therefore \omega^2=\frac{2 k}{I}=2 \mathrm{k} \cdot \frac{2}{\mathrm{md}^2}=\frac{4 \mathrm{k}}{\mathrm{md}^2}$$

$$\therefore \omega=\frac{2}{\mathrm{~d}} \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$$

Question100

A body of mass 'm' and radius of gyration 'K' has an angular momentum L. Its angular velocity is

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Options:

A. $\frac{K^2}{mL}$

B. $mK^2 L$

C. $\frac{mK^2}{L}$

D. $\frac{L}{mK^2}$

Answer: D

Solution:

$$L = I\omega = mk^2\omega$$
$$\therefore \omega = \frac{L}{mk^2}$$

Question101

The moment of inertia of a body about the given axis, rotating with angular velocity 1 rad/s is numerically equal to 'P' times its rotational kinetic energy. The value of 'P' is

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Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 2

D. 1

Answer: C

Solution:

The angular velocity

$$\omega = 1 \text{ rad/s}$$
$$I = P \cdot \left(\frac{1}{2} \mathrm{I}\omega^2\right)$$
$$1 = \mathrm{P} \cdot \frac{1}{2} \cdot 1$$
$$\therefore \quad P = 2$$

Question102

Two circular loops P and Q are made from a uniform wire. The radii of P and Q are R_1 and R_2 respectively. The momentsw of inertia about their own axis are I_P and I_Q respectively. If $\frac{\mathrm{I}_\mathrm{P}}{\mathrm{I}_Q} = \frac{1}{8}$ then $\frac{\mathrm{R}_2}{\mathrm{R}_1}$ is

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Options:

A. 4

B. 3

C. 2

D. 5

Answer: C

Solution:

To solve this problem, let's start by understanding how the moment of inertia works for a circular loop. For a circular loop made from a uniform wire of radius R and mass M, the moment of inertia about its own axis can be given by:

$\mathrm{I} = MR^2$

Given two loops P and Q with radii R_1 and R_2 respectively, and their moments of inertia I_P and I_Q, we have the following relationship:

$\frac{I_P}{I_Q} = \frac{1}{8}$

Using the formula for the moment of inertia, we can write:

$I_P = M_1 R_1^2$

$I_Q = M_2 R_2^2$

Since the loops are made of the same uniform wire, the mass (which is proportional to the length of the wire used) will be different for different radii. The mass of a loop is proportional to the circumference so:

$M_1 \propto 2\pi R_1$

$M_2 \propto 2\pi R_2$

We can express the masses in terms of a common unit length mass λ:

$M_1 = \lambda(2\pi R_1)$

$M_2 = \lambda(2\pi R_2)$

Substitute these into the moments of inertia:

$I_P = \lambda(2\pi R_1) R_1^2 = 2\pi\lambda R_1^3$

$I_Q = \lambda(2\pi R_2) R_2^2 = 2\pi\lambda R_2^3$

Using the given ratio:

$\frac{I_P}{I_Q} = \frac{2\pi\lambda R_1^3}{2\pi\lambda R_2^3} = \frac{R_1^3}{R_2^3}$

Given that:

$\frac{R_1^3}{R_2^3} = \frac{1}{8}$

We take the cube root of both sides:

$\frac{R_1}{R_2} = \sqrt[3]{\frac{1}{8}}$

Simplify the cube root:

$\frac{R_1}{R_2} = \frac{1}{2}$

Therefore:

$\frac{R_2}{R_1} = 2$

So the correct answer is **Option C: 2**.

Question103

A metre scale is supported on a wedge at its centre of gravity. A body of weight 'w'. is suspended from the $20 \mathrm{~cm}$ mark and another weight of 25 gram is suspended from $74 \mathrm{~cm}$ mark balance it and the metre scale remains perfectly horizontal. Neglecting the weight of the metre scale, the weight of the body is

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Options:

A. 20 gram-wt

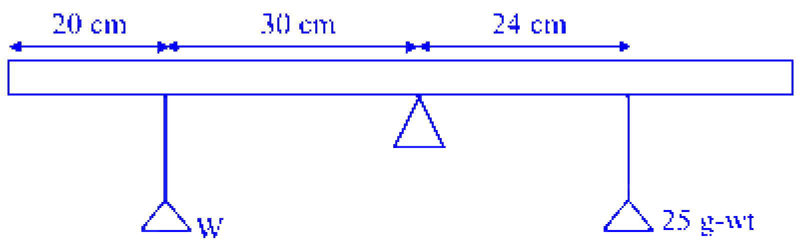
B. 15 gram-wt

C. 33 gram-wt

D. 30 gram-wt

Answer: A

Solution:



Taking moments about the centre of the scale we get

$$W \times 30 = 25 \times 24$$
$$\therefore W = \frac{25 \times 24}{30} = 20 \mathrm{~g}-\mathrm{wt}$$

Question104

A body of mass 'm' and radius of gyration 'K' has an angular momentum 'L'. Then its angular velocity is

Options:

A. $\frac{\mathrm{L}}{\mathrm{mK}^2}$

B. $\frac{\mathrm{mK}^2}{\mathrm{L}}$

C. $\frac{\mathrm{K}^2}{\mathrm{mL}}$

D. $\mathrm{mK}^2\ \mathrm{L}$

Answer: A

Solution:

To determine the angular velocity of a body with a given mass 'm', radius of gyration 'K', and angular momentum 'L', we need to use the relationship between these quantities. The moment of inertia 'I' of the body is given by:

$I = mK^2$

Angular momentum 'L' is also related to the moment of inertia 'I' and angular velocity 'ω' by the following equation:

$L = I\omega$

Substituting the expression for 'I' into the equation for 'L' gives:

$L = (mK^2)\omega$

To solve for the angular velocity 'ω', we can rearrange this equation:

$\omega = \frac{L}{mK^2}$

Therefore, the correct option for the angular velocity is:

Option A

$\frac{\mathrm{L}}{\mathrm{mK}^2}$

Question105

A molecule consists of two atoms each of mass 'm' and separated by a distance 'd'. At room temperature, if the average rotational kinetic energy is 'E' then the angular frequency is

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Options:

A. $\frac{2}{\mathrm{d}}\sqrt{\frac{\mathrm{E}}{\mathrm{m}}}$

B. $\frac{\mathrm{d}}{2}\sqrt{\frac{\mathrm{m}}{\mathrm{E}}}$

C. $\sqrt{\frac{Ed}{m}}$

D. $\sqrt{\frac{m}{Ed}}$

Answer: A

Solution:

To determine the angular frequency of a molecule consisting of two atoms, we need to use the concept of rotational kinetic energy. The average rotational kinetic energy of a diatomic molecule at room temperature is given by:

$E = \frac{1}{2}I\omega^2$

Here, E is the average rotational kinetic energy, I is the moment of inertia, and ω is the angular frequency.

The moment of inertia for the system can be calculated using the formula:

$I = \mu d^2$

where μ is the reduced mass of the two-atom system and d is the separation distance. For two atoms of mass m, the reduced mass μ is given by:

$\mu = \frac{m \cdot m}{m+m} = \frac{m^2}{2m} = \frac{m}{2}$

Thus, the moment of inertia becomes:

$I = \frac{m}{2}d^2$

Substituting I back into the rotational kinetic energy equation, we get:

$E = \frac{1}{2}\left(\frac{m}{2}d^2\right)\omega^2$

Simplifying this, we obtain:

$E = \frac{md^2\omega^2}{4}$

To solve for the angular frequency ω, we rearrange the equation:

$\omega^2 = \frac{4E}{md^2}$

Taking the square root of both sides, we find:

$\omega = \sqrt{\frac{4E}{md^2}}$

Which simplifies to:

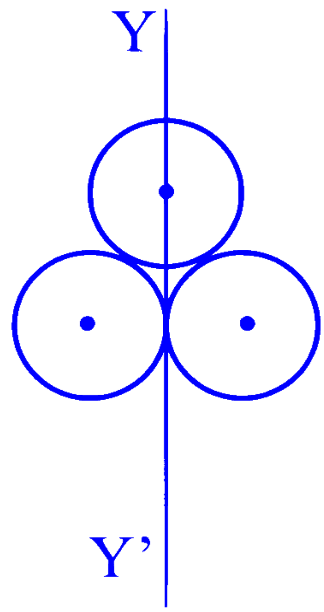
$\omega = \frac{2}{d}\sqrt{\frac{E}{m}}$

Thus, the correct answer is:

Option A: $\frac{2}{\mathrm{d}}\sqrt{\frac{\mathrm{E}}{\mathrm{m}}}$

Question106

Three solid spheres each of mass 'M' and radius 'R' are arranged as shown in the figure. The moment of inertia of the system about YY' will be



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Options:

A. $\frac{16}{5}\mathrm{MR}^2$

B. $\frac{21}{5}\mathrm{MR}^2$

C. $\frac{7}{5}\mathrm{MR}^2$

D. $\frac{11}{5}\mathrm{MR}^2$

Answer: A

Solution:

Moment of inertia of the upper sphere $= \frac{2}{5}\mathrm{MR}^2$

For each lower sphere M.I. $= \frac{2}{5}\mathrm{MR}^2 + \mathrm{MR}^2 = \frac{7}{5}\mathrm{MR}^2$

$\therefore$ Total moment of inertia $\mathrm{I} = \frac{2}{5}\mathrm{MR}^2 + 2 \times \frac{7}{5}\mathrm{MR}^2 = \frac{16}{5}\mathrm{MR}^2$

Question107

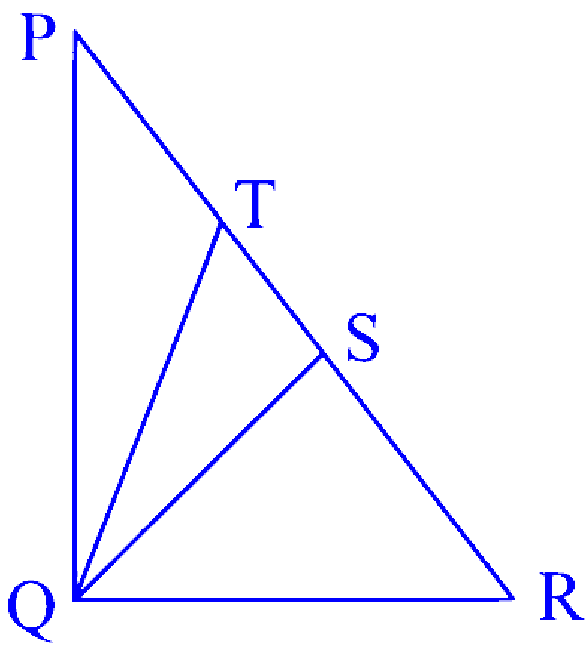

Figure shows triangular lamina which can rotate about different axes moment of inertia is maximum, about the axis

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Options:

A. PR

B. QS

C. QR

D. PQ

Answer: C

Solution:

The moment of inertia is given by

$I = \sum m_i r_i^2$

If the distance of particles from the axis of rotation is larger, then the moment of inertia will be larger. For axis QR, the particles will be situated at greater distances and hence the moment of inertia will be greater.

Question108

A particle with position vector $\overrightarrow{\mathrm{r}}$ has a linear momentum $\overrightarrow{\mathrm{P}}$. Which one of the following statements is true in respect of its angular momentum 'L' about the origin?

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Options:

A. $\overrightarrow{\mathrm{L}}$ acts along $\overrightarrow{\mathrm{P}}$.

B. L is maximum when $\overrightarrow{\mathrm{P}}$ is perpendicular to $\overrightarrow{\mathrm{r}}$.

C. $\overrightarrow{\mathrm{L}}$ acts along $\overrightarrow{\mathrm{r}}$.

D. L is maximum when $\overrightarrow{\mathrm{P}}$ and $\overrightarrow{\mathrm{r}}$ are parallel.

Answer: B

Solution:

The angular momentum of the particle is given by $\overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{r}} \times \overrightarrow{\mathrm{p}}$

Its magnitude is $\mathrm{L}=\mathrm{rp} \sin \theta$

L will be maximum when $\theta=90^{\circ}$.

Question109

A child is standing with folded hands at the centre of the platform rotating about its central axis. The kinetic energy of the system is 'K'. The child now stretches his arms so that the moment of inertia of the system becomes double. The kinetic energy of the system now is

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Options:

A. $\frac{K}{2}$

B. 2 K

C. 4 K

D. $\frac{\mathrm{K}}{4}$

Answer: A

Solution:

To answer this question, we need to understand the relationship between moment of inertia, angular momentum, and kinetic energy in the context of rotational motion. The conservation of angular momentum plays a crucial role in this problem.

First, we know that the angular momentum (L) is conserved when there is no external torque acting on the system. Angular momentum can be defined as the product of the moment of inertia (I) and the angular velocity (ω), given by the formula:

$L=I \omega$

Given in the question, when the child stretches his arms out, the moment of inertia of the system becomes double. If we let the initial moment of inertia be I, then the final moment of inertia after the child stretches his arms becomes $2 I$.

Since angular momentum is conserved and initially, the system had a certain angular momentum $L = I\omega$, after the change, it still maintains the same angular momentum, now with the new moment of inertia, leading to a new angular velocity ω' such that:

$L = I\omega = 2I\omega'$

From this equation, we solve for ω' to find that:

$\omega' = \frac{\omega}{2}$

This indicates that the angular velocity is halved when the moment of inertia is doubled.

The kinetic energy (K) of a rotating body is given by:

$K = \frac{1}{2}I\omega^2$

Initially, the kinetic energy is:

$K = \frac{1}{2}I\omega^2$

After the moment of inertia is doubled and the angular velocity is halved:

$K' = \frac{1}{2}(2I)\left(\frac{\omega}{2}\right)^2$

$K' = \frac{1}{2} \cdot 2I \cdot \frac{\omega^2}{4}$

$K' = \frac{1}{2}I\omega^2 \cdot \frac{1}{2}$

$K' = \frac{1}{2}K$

Therefore, the kinetic energy of the system after the child stretches his arms out, resulting in the moment of inertia becoming double, is $\frac{K}{2}$.

The correct answer is **Option A** $\frac{K}{2}$.

Question110

Two rings of radius 'R' and 'nR' made of same material have the ratio of moment of inertia about an axis passing through its centre and perpendicular to the plane is $1 : 8$. The value of 'n' is (mass per unit length $= \lambda$)

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Options:

A. 2

B. 4

C. 1

D. 3

Answer: A

Solution:

To find the value of n given the two rings of radius 'R' and 'nR' and knowing that the ratio of their moments of inertia is 1 : 8, we first establish the formula for the moment of inertia of a ring about an axis passing through its centre and perpendicular to the plane of the ring.

The moment of inertia (I) of a ring about an axis passing through its centre and perpendicular to the plane of the ring is given by the formula:

$I = mR^2$

where m is the mass of the ring and R is its radius.

Given the mass per unit length λ, the mass of the first ring with radius R can be expressed as:

$m_1 = \lambda \cdot 2\pi R$

The moment of inertia of the first ring, therefore, is:

$I_1 = (\lambda \cdot 2\pi R) \cdot R^2 = 2\pi\lambda R^3$

The second ring has a radius of nR. Therefore, its mass m_2 is:

$m_2 = \lambda \cdot 2\pi(nR)$

And the moment of inertia of the second ring is:

$I_2 = (\lambda \cdot 2\pi(nR)) \cdot (nR)^2 = 2\pi\lambda n^3 R^3$

It's given that the ratio of their moments of inertia is 1 : 8, so:

$\frac{I_1}{I_2} = \frac{1}{8}$

Substituting the expressions for I_1 and I_2, we get:

$\frac{2\pi\lambda R^3}{2\pi\lambda n^3 R^3} = \frac{1}{8}$

Simplifying this expression, we see that π, λ, and R^3 cancel out, leaving us with:

$\frac{1}{n^3} = \frac{1}{8}$

Solving for n, we find:

$n^3 = 8$

Taking the cube root of both sides gives us:

$n = 2$

Therefore, the value of n is 2, which corresponds to **Option A**.

Question111

Two rotating bodies P and Q of masses $'\mathrm{m}'$ and $'2\ \mathrm{m}'$ with moment of inertia I_P and I_Q $(I_Q > I_P)$ have equal Kinetic energy of rotation. If L_P and L_Q be their angular momenta respectively then

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Options:

A. $\mathrm{L}_{\mathrm{Q}} = 0$

B. $\mathrm{L}_Q = \mathrm{L}_P$

C. $\mathrm{L}_Q < \mathrm{L}_{\mathrm{P}}$

D. $\mathrm{L}_Q > \mathrm{L}_{\mathrm{P}}$

Answer: D

Solution:

Here's a step-by-step explanation to determine the relationship between the angular momenta of the two rotating bodies:

Understanding the Concepts

- **Rotational Kinetic Energy:** The kinetic energy of a rotating body is given by:

$K.E. = \frac{1}{2} I\omega^2$

where:

- I is the moment of inertia
- ω is the angular velocity
- **Angular Momentum:** The angular momentum of a rotating body is given by:

$L = I\omega$

Analyzing the Problem

1. **Equal Kinetic Energy:** The problem states that the kinetic energies of the two bodies are equal:

$\frac{1}{2}I_P\omega_P^2 = \frac{1}{2}I_Q\omega_Q^2$

1. **Moment of Inertia Relationship:** We are given that $I_Q > I_P$.
2. **Solving for Angular Velocities:** From the equal kinetic energy equation, we can see that:

$\omega_Q^2 = \frac{I_P}{I_Q}\omega_P^2$

Since $I_Q > I_P$, this implies $\omega_Q^2 < \omega_P^2$, and therefore $\omega_Q < \omega_P$.

1. **Comparing Angular Momenta:** Now let's compare the angular momenta:

- $L_P = I_P\omega_P$
- $L_Q = I_Q\omega_Q$

Since $I_Q > I_P$ and $\omega_Q < \omega_P$, the overall effect is that $L_Q > L_P$.

Conclusion

Therefore, the correct answer is **Option D:** $L_Q > L_P$.

Question112

A solid sphere of mass 'M' and radius 'R' is rotating about its diameter. A solid cylinder of same mass and same radius is also rotating about its geometrical axis with an angular speet twice that of the sphere. The ratio of the kinetic energy of rotation of the sphere to that of the cylinder is

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Options:

A. 2 : 3

B. 1 : 5

C. 1 : 4

D. 3 : 1

Answer: B

Solution:

To find the ratio of the kinetic energy of rotation of the sphere to that of the cylinder, we need to use the formulas for the rotational kinetic energy and the moments of inertia for both shapes.

The kinetic energy of rotation is given by:

$$KE = \frac{1}{2} I \omega^2$$

where I is the moment of inertia and ω is the angular speed.

For a solid sphere rotating about its diameter, the moment of inertia is:

$$I_{\text{sphere}} = \frac{2}{5} MR^2$$

For a solid cylinder rotating about its geometrical axis, the moment of inertia is:

$$I_{\text{cylinder}} = \frac{1}{2} MR^2$$

Given that the angular speed of the cylinder is twice that of the sphere, if we let the angular speed of the sphere be $\omega_{\text{sphere}} = \omega$, then the angular speed of the cylinder will be $\omega_{\text{cylinder}} = 2\omega$.

Now, we can write the rotational kinetic energies:

For the sphere:

$$KE_{\text{sphere}} = \frac{1}{2} I_{\text{sphere}} \omega^2 = \frac{1}{2} \left(\frac{2}{5} MR^2\right) \omega^2 = \frac{1}{5} MR^2 \omega^2$$

For the cylinder:

$$KE_{\text{cylinder}} = \frac{1}{2} I_{\text{cylinder}} (2\omega)^2 = \frac{1}{2} \left(\frac{1}{2} MR^2\right) (2\omega)^2 = \frac{1}{2} \left(\frac{1}{2} MR^2\right) 4\omega^2 = MR^2 \omega^2$$

So, the ratio of the kinetic energy of rotation of the sphere to that of the cylinder is:

$$\text{Ratio} = \frac{KE_{\text{sphere}}}{KE_{\text{cylinder}}} = \frac{\frac{1}{5}MR^2\omega^2}{MR^2\omega^2} = \frac{1}{5}$$

Therefore, the correct answer is:

Option B: 1 : 5

Question113

A particle performs rotational motion with an angular momentum 'L'. If frequency of rotation is doubled and its kinetic energy becomes one fourth, the angular momentum becomes.

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Options:

A. L

B. $\frac{L}{4}$

C. $\frac{L}{8}$

D. $\frac{L}{2}$

Answer: C

Solution:

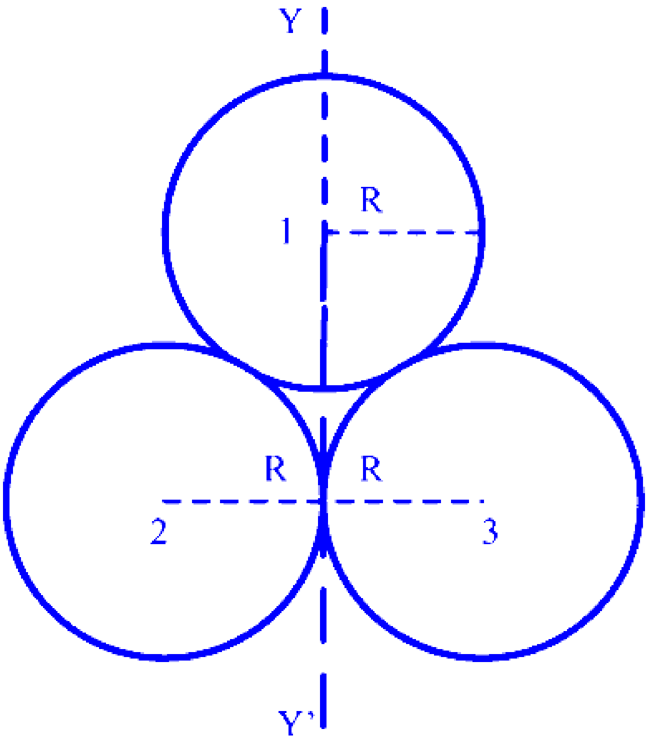
Kinetic energy $k = \frac{1}{2}I\omega^2$

$$\therefore \frac{K_2}{K_1} = \frac{I_2\omega_2^2}{I_1\omega_1^2} \quad \therefore \frac{1}{4} = \frac{I_2}{I_1} \cdot 4 \quad \therefore \frac{I_2}{I_1} = \frac{1}{16}$$

$$\frac{L_2}{L_1} = \frac{I_2\omega_2}{I_1\omega_1} = \frac{1}{16} \times 2 = \frac{1}{8}$$

$$\therefore L_2 = \frac{L_1}{8}$$

Question114



Three rings each of mass 'M' and radius 'R' are arranged as shown in the figure. The moment of inertia of system about axis YY' will be

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Options:

A. $5 \mathrm{MR}^2$

B. $\frac{7}{2} \mathrm{MR}^2$

C. $\frac{3}{2} \mathrm{MR}^2$

D. $3 \mathrm{MR}^2$

Answer: B

Solution:

The moment of inertia of the upper ring about its diameter is given by

$I_1 = \frac{MR^2}{2}$

The moment of inertia of the two lower rings about a tangent in their plane is given by

$I_2 = I_3 = \frac{3}{2}MR^2$

Total moment of inertia

$I = I_1 + I_2 + I_3$

$= \frac{MR^2}{2} + \frac{3}{2}MR^2 + \frac{3}{2}MR^2 = \frac{7}{2}MR^2$

Question115

The moment of inertia of a circular disc of radius $2\ \mathrm{m}$ and mass $1\ \mathrm{kg}$ about an axis XY passing through its centre of mass and perpendicular to the plane of the disc is $2\ \mathrm{kg\ m}^2$. The moment of inertia about an axis parallel to the axis XY and passing through the edge of the disc is

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Options:

A. $6\ \mathrm{kg\ m}^2$

B. $4\ \mathrm{kg\ m}^2$

C. $10\ \mathrm{kg\ m}^2$

D. $8\ \mathrm{kg\ m}^2$

Answer: A

Solution:

Step 1: Use the parallel axis theorem

The parallel axis theorem states that the moment of inertia about any axis ($\mathbf{I}$) is the moment of inertia about a parallel axis through the center of mass ($\mathbf{I_{cm}}$) plus the product of the mass ($\mathbf{M}$) and the square of the distance between the two axes ($\mathbf{d^2}$). The distance between the center of the disc and its edge is the radius ($\mathbf{R}$).

$$\mathbf{I} = \mathbf{I_{cm}} + \mathbf{Md}^2 = \mathbf{I_{cm}} + \mathbf{MR}^2$$

Step 2: Substitute the given values

The given values are:

- $\mathbf{I_{cm}} = 2 \text{ kg m}^2$
- $\mathbf{M} = 1 \text{ kg}$
- $\mathbf{R} = 2 \text{ m}$

Substitute these values into the equation:

$$\mathbf{I} = 2 \text{ kg m}^2 + (1 \text{ kg})(2 \text{ m})^2$$
$$\mathbf{I} = 2 \text{ kg m}^2 + (1 \text{ kg})(4 \text{ m}^2)$$
$$\mathbf{I} = 2 \text{ kg m}^2 + 4 \text{ kg m}^2$$
$$\mathbf{I} = 6 \text{ kg m}^2$$

Answer:

(A) **6 kg m²**

Question116

The moment of inertia of a body about a given axis is 1.2 kg/m^3. Initially the body is at rest. In order to produce rotational kinetic energy of 1500 J, an angular acceleration of 25rad/s^2 must be applied about an axis for a time duration of

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Options:

A. 8 s

B. 2 s

C. 4 s

D. 1 s

Answer: B

Solution:

$$\mathrm{I}=1.2\ \mathrm{kg}-\mathrm{m}^2, \quad \text{K.E.} = 1500\ \mathrm{J}, \alpha = 25 \mathrm{rad/s}^2$$

$$\text{K.E.} = \frac{1}{2}\mathrm{I}\omega^2$$

$$\therefore 1500 = \frac{1}{2} \times 1.2 \times \omega^2$$

$$\therefore \omega^2 = \frac{2 \times 1500}{1.2} = 2500 \quad \therefore \omega = 50 \mathrm{rad/s}$$

$$\omega = \omega_0 + \alpha \mathrm{t}$$

$$\therefore 50 = 0 + 25\mathrm{t} = 25\mathrm{t} \quad \therefore \mathrm{t} = 2\ \mathrm{s}$$

Question117

A disc of radius 0.4 metre and mass 1 kg rotates about an axis passing through its centre and perpendicular to its plane. The angular acceleration is 10 rad s^{-2}. The tangential force applied to the rim of the disc is

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Options:

A. 2N

B. 3N

C. 4N

D. 5N

Answer: A

Solution:

$\mathrm{R}=0.4 \mathrm{~m}, \mathrm{M}=1 \mathrm{~kg}, \alpha=10 \mathrm{rad} / \mathrm{s}^2$

$I=\frac{M^2}{2}=\frac{1 \times(0.4)^2}{2}=0.08 \mathrm{~kg} \mathrm{~m}^2$

Torque, $\tau=\mathrm{I} \alpha=0.08 \times 10=0.8 \mathrm{~N}-\mathrm{m}$

Also, $\tau=F R$

or $F=\frac{\tau}{R}=\frac{0.8}{0.4} \quad=2 \mathrm{~N}$

Question118

The ratio of radii of gyration of a circular ring and circular disc of the same mass and radius, about an axis passing through their centres and perpendicular to their planes is

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Options:

A. $1: \sqrt{2}$

B. $2: 1$

C. $\sqrt{2}: 1$

D. $3: 2$

Answer: C

Solution:

$\mathrm{I}=\mathrm{MK}^2$

For a ring, $\quad \mathrm{MR}^2=\mathrm{MK}_{\mathrm{r}}^2 \quad \therefore \mathrm{K}_{\mathrm{r}}=\mathrm{R}$

For a disc, $\quad \frac{\mathrm{MR}^2}{2}=\mathrm{MK}_{\mathrm{d}}^2 \quad \therefore \mathrm{K}_{\mathrm{d}}=\frac{\mathrm{R}}{\sqrt{2}}$

$\therefore \frac{\mathrm{K}_{\mathrm{r}}}{\mathrm{K}_{\mathrm{d}}}=\sqrt{2}$

Question119

A solid sphere of mass M, radius R has moment of inertia 'I' about its diameter. It is recast into a disc of thickness 't' whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains 'I'. Radius of the disc will be

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Options:

A. $\frac{4R}{\sqrt{11}}$

B. $\frac{3R}{4}$

C. $\frac{2R}{\sqrt{15}}$

D. $\frac{2R}{3}$

Answer: C

Solution:

Moment of inertia of the solid sphere $\mathrm{I} = \frac{2}{5}\mathrm{MR}^2$

Moment of inertia of the disc about an axis passing through its edge and perpendicular to the plane is given by

$$\mathrm{I}' = \frac{\mathrm{MR}'^2}{2} + \mathrm{MR}'^2 = \frac{3}{2}\mathrm{MR}'^2$$

$$\mathrm{I}' = \mathrm{I} \quad \therefore \frac{3}{2}\mathrm{MR}'^2 = \frac{2}{5}\mathrm{MR}^2$$

$$\therefore \quad \mathrm{R}' = \frac{4}{15}\mathrm{R}^2 \quad \therefore \mathrm{R}' = \frac{2}{\sqrt{15}}\mathrm{R}$$

Question120

Two bodies rotate with kinetic energies 'E_1' and 'E_2'. Moments of inertia about their axis of rotation are 'I_1' and 'I_2'. If $I_1 = \frac{I_2}{3}$ and E_1

= 27 E_2, then the ratio of angular momenta 'L_1' to 'L_2' is

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Options:

A. 1 : 3

B. 3 : 1

C. 1 : 1

D. 2 : 1

Answer: B

Solution:

$$
\begin{aligned}
& \mathrm{E}_1=\frac{1}{2} \mathrm{I}_1 \omega_1^2 \\
& \mathrm{E}_2=\frac{1}{2} \mathrm{I}_2 \omega_2^2=\frac{1}{2} \cdot 3 \mathrm{I}_1 \cdot \omega_1^2 \quad\left[\because \mathrm{I}_2=3 \mathrm{I}_1\right] \\
& =\frac{3}{2} \mathrm{I}_1 \omega_2^2 \\
& \mathrm{E}_1=27 \mathrm{E}_2=27 \times \frac{3}{2} \times \mathrm{I}_1 \omega_2^2
\end{aligned}
$$

$$
\begin{aligned}
& \therefore \frac{1}{2} \mathrm{I}_1 \omega_2^2=\frac{81}{2} \mathrm{I}_1 \omega_2^2 \\
& \therefore \quad \omega_1=9 \omega_2 \\
& \frac{L_1}{L_2}=\frac{\mathrm{I}_1 \omega_1}{\mathrm{I}_2 \omega_2}=\frac{\mathrm{I}_1 \times 9 \omega_2}{3 \mathrm{I}_1 \times \omega_2}=3
\end{aligned}
$$

Question121

A disc of radius 0.4 m and mass one kg rotates about an axis passing through its centre and perpendicular to its plane. The angular acceleration of the disc is 10 rad/s^2. The tangential force applied to the rim of the disc is

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Options:

A. 4 N

B. 1 N

C. 2 N

D. 8 N

Answer: C

Solution:

$\mathrm{R}=0.4\ \mathrm{m}, \mathrm{M}=1\ \mathrm{kg}, \alpha=10\ \mathrm{rad}/\mathrm{s}^2$

Moment of inertia $\mathrm{I}=\frac{\mathrm{MR}^2}{2}=\frac{1\times(0.4)^2}{2}=\frac{0.16}{2}=0.08\ \mathrm{kg}\ \mathrm{m}^2$

Torque

$$\tau=\mathrm{RF}=\mathrm{I}\alpha$$
$$\therefore \mathrm{F}=\frac{\mathrm{I}\alpha}{\mathrm{R}}=\frac{0.08\times 10}{0.4}=2\ \mathrm{N}$$

Question122

Three points masses, each of mass m are placed at the corners of an equilateral triangle of side ℓ. The moment of inertia of the system about an axis passing through one of the vertices and parallel to the side joining other two vertices, will be

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Options:

A. $\frac{3}{4}m\ell^2$

B. $\frac{1}{4}m\ell^2$

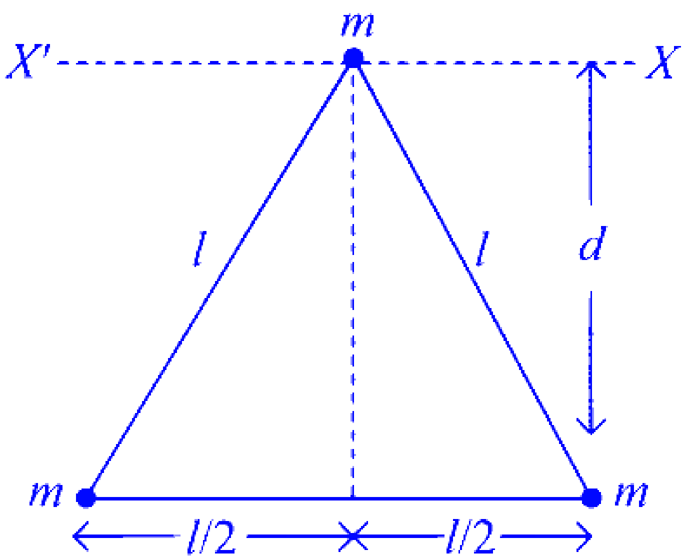
C. $\frac{3}{2}m\ell^2$

D. $\frac{1}{2}m\ell^2$

Answer: C

Solution:

The given situation is as shown below



We have to find the moment of inertia about axis XX'.

The distance of line XX' from base of triangle is

$d = \sqrt{I^2 - \frac{l^2}{4}} = \frac{\sqrt{3}I}{2}$

$\therefore$ Moment of inertia, due to each mass,

$$I = m \times 0 + m\left(\frac{\sqrt{3}}{2}I\right)^2 + m\left(\frac{\sqrt{3}}{2}I\right)^2$$
$$= \frac{3ml^2}{4} + \frac{3ml^2}{4}$$
$$= \frac{6ml^2}{4} = \frac{3ml^2}{2}$$

Question123

A rope is wound around a solid cylinder of mass 1 kg and radius 0.4 m . What is the angular acceleration of cylinder, if the rope is pulled with a force of 25 N ? (Cylinder is rotating about its own axis.)

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Options:

A. 125 rad/s^2

B. 10 rad/s^2

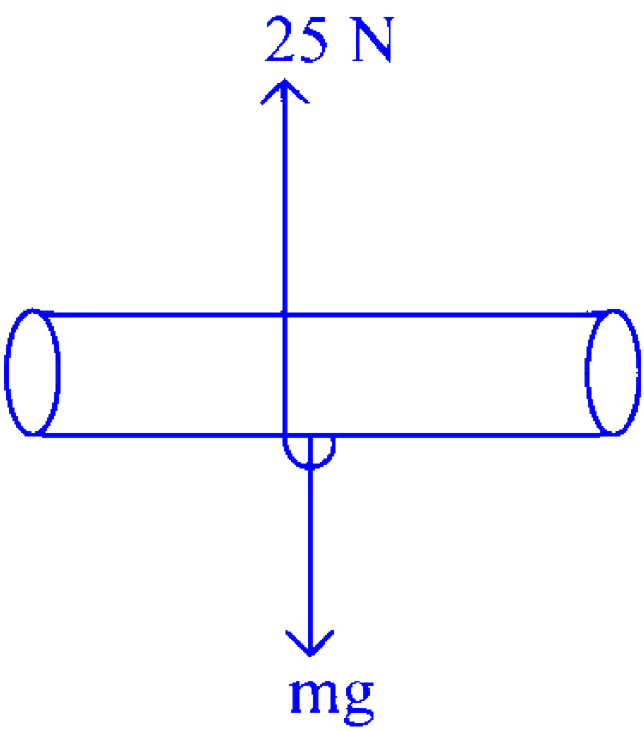
C. 1 rad/s^2

D. 50 rad/s^2

Answer: A

Solution:

Consider the free body diagram shown below,



,

The torque about centre of cylinder,

$\tau = F \times r = 25 \times 0.4 = 10\text{Nm}$

The relation between angular acceleration (α) and torque (τ) is given as

$$\begin{aligned} \tau &= l\alpha \\ &= \frac{Mr^2}{2}\alpha \quad (\text{ for solid cylinder, } I = \frac{MR^2}{2}) \\ 10 &= \frac{1 \times (0.4)^2}{2}\alpha \Rightarrow \alpha = 125 \mathrm{rad/s^2} \end{aligned}$$

Question124

Two circular loop A and B of radii R and NR respectively are made from a uniform wire. Moment of inertia of B about its axis is 3 times that of A about its axis. The value of N is

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Options:

A. $[2]^{\frac{1}{3}}$

B. $[5]^{\frac{1}{3}}$

C. $[3]^{\frac{1}{3}}$

D. $[4]^{\frac{1}{3}}$

Answer: C

Solution:

Given, $r_A = R$ and $r_B = NR$

Mass of wire $A, m_A = 2\pi r_A \rho = 2\pi R\rho$

Mass of wire $B, m_B = 2\pi r_B \rho = 2\pi NR\rho = Nm_A$

The moment of inertia of a circular loop is

$I = mr^2$

$$\begin{aligned} &\text{Here,} \quad I_B = 3I_A \\ &m_B r_B^2 = 3 \times m_A r_A^2 \\ &\Rightarrow \quad Nm_A(NR)^2 = 3m_A(R)^2 \\ &\Rightarrow \quad N^3 = 3 \text{ or } N = (3)^{1/3} \end{aligned}$$

Question125

A body slides down a smooth inclined plane having angle θ and reaches the bottom with velocity v. If a body is a sphere, then its linear velocity at the bottom of the plane is

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Options:

A. $\sqrt{\frac{9}{7}}v$

B. $\sqrt{\frac{5}{7}}v$

C. $\sqrt{\frac{2}{7}}v$

D. $\sqrt{\frac{3}{7}}v$

Answer: B

Solution:

The linear velocity of the body, $v=\sqrt{2gh}$

The velocity of the sphere about its centre,

$$v_{\mathrm{CM}}=\sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}=\frac{v}{\sqrt{1+\frac{K^2}{R^2}}} \quad \text{... (i)}$$

For uniform solid sphere,

$$\frac{K^2}{R^2}=\frac{2}{5}$$

Substituting value in Eq. (i), we get

$$v_{\mathrm{CM}}=\frac{v}{\sqrt{1+\left(\frac{2}{5}\right)}}=\sqrt{\frac{5}{7}}v$$

Question126

A thin uniform rod has mass M and length L The moment of inertia about an axis perpendicular to it and passing through the point at a distance $\frac{L}{3}$ from one of its ends, will be

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Options:

A. $\frac{ML^2}{12}$

B. $\frac{7}{8}ML^2$

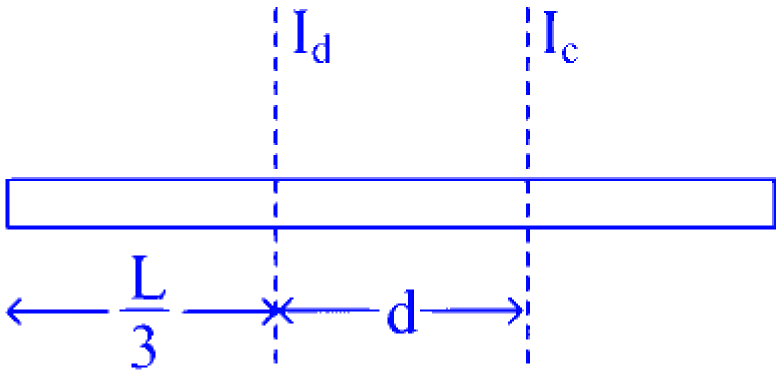
C. $\frac{ML^2}{9}$

D. $\frac{ML^2}{3}$

Answer: C

Solution:

Consider the figure shown below



A point at a distance $\frac{L}{3}$ from one end is at a distance of $d = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$ from the centre of rod.

The moment of inertia of rod about an axis passing through centre and perpendicular to it is

$I_c = \frac{ML^2}{12}$

Using parallel axis theorem,

$$I_d = I_c + Md^2$$
$$= \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$$

Question127

The ratio of radii of gyration of a ring to a disc (both circular) of same radii and mass, about a tangential axis perpendicular to the plane is

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Options:

A. $\frac{\sqrt{3}}{\sqrt{2}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{\sqrt{2}}{1}$

D. $\frac{2}{\sqrt{3}}$

Answer: D

Solution:

The radius of gyration is given by

$$K = \sqrt{\frac{I}{M}} \Rightarrow K \propto \sqrt{1}$$

Moment of inertia of ring about tangential axis is calculated by parallel axis theorem,

$$I_{\text{ring}} = I_c + MR^2 = MR^2 + MR^2 = 2MR^2$$

Similarly,

$$I_{\text{disc}} = I_c + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$\therefore \quad \frac{K_{\text{ring}}}{K_{\text{disc}}} = \sqrt{\frac{I_{\text{ring}}}{I_{\text{disc}}}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

Question128

If there is a change of angular momentum from 1j-s to 4j-s in 4 s, then the torque

Options:

A. $\left(\frac{5}{4}\right)$J

B. $\left(\frac{3}{4}\right)$J

C. 1 J

D. $\left(\frac{4}{3}\right)$J

Answer: B

Solution:

The torque (τ) can be found by using the relation between torque and the rate of change of angular momentum. This relationship is given by:

$\tau=\frac{\Delta L}{\Delta t}$

where:

- ΔL is the change in angular momentum,
- Δt is the change in time.

In this case, the change in angular momentum (ΔL) from 1j s to 4j s is :

$\Delta L=4\,\mathrm{j\,s}-1\,\mathrm{j\,s}=3\,\mathrm{j\,s}$

And the change in time (Δt) is 4 s.

Therefore, the torque can be calculated as:

$\tau=\frac{3\,\mathrm{j\,s}}{4\,\mathrm{s}}=\frac{3}{4}$ J

Thus, the correct answer is **Option B**, $\left(\frac{3}{4}\right)$ J.

Question129

A solid cylinder of radius r and mass M rolls down an inclined plane of height h. When it reaches the bottom of the plane, then its rotational kinetic energy is ($g=$ acceleration due to gravity)

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Options:

A. $\frac{Mgh}{4}$

B. $\frac{Mgh}{2}$

C. Mgh

D. $\frac{Mgh}{3}$

Answer: D

Solution:

When the solid cylinder reaches the bottom of the plane, its rotational kinetic energy is given by

$K_r = \frac{1}{2}\omega^2$

As, $I = Mk^2$ and $\omega = \frac{V_{\text{CM}}}{R}$

$\Rightarrow K_r = \frac{1}{2}Mk^2 \times \left(\frac{v_{\text{CM}}}{R^2}\right)^2 = \frac{1}{2}Mv_{\text{CM}}^2\left(\frac{k^2}{R^2}\right)$

At bottom, velocity, $v_{\text{CM}} = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$

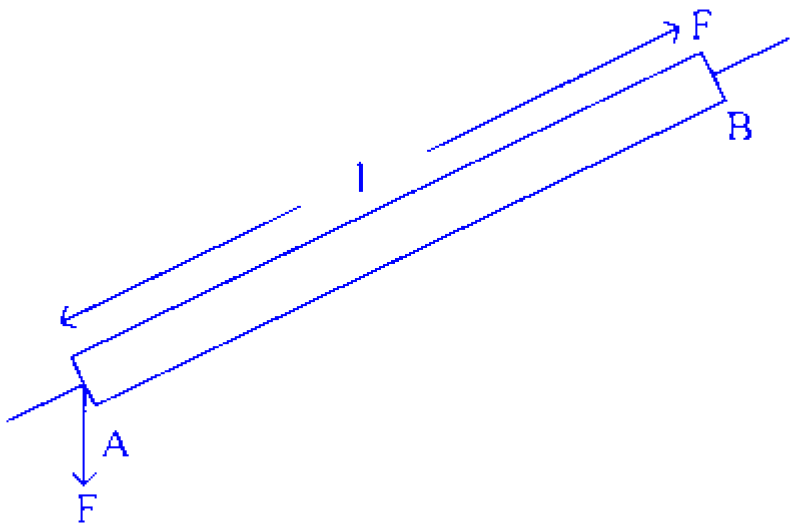
Also, for solid cylinder, $\frac{k^2}{R^2} = \frac{1}{2}$

$\therefore K_r = \frac{1}{2}M\left[\frac{2gh}{\left(1+\frac{1}{2}\right)}\right]\left(\frac{1}{2}\right) = \frac{4Mgh}{4\times 3} = \frac{Mgh}{3}$

Question130

A rod l m long is acted upon by a couple as shown in the figure. The moment of couple is τ Nm. If the force at each end of the rod, then magnitude of each force is

$(\sin 30^\circ = \cos 60^\circ = 0.5)$



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Options:

A. $\frac{\tau}{l}$

B. $\frac{l}{2\tau}$

C. $\frac{2\tau}{l}$

D. $\frac{2}{\tau}$

Answer: C

Solution:

Length of rod $= l$

Moment of couple,

$$\tau = Fl\sin\theta$$
$$\therefore F = \frac{\tau}{l\sin\theta} = \frac{\tau}{l\sin 30^\circ} = \frac{\tau}{l \times \frac{1}{2}}$$

Force, $\quad F = \frac{2\tau}{l}$

Question131

A solid sphere rolls down from top of inclined plane, 7 m high, without slipping. Its linear speed at the foot of plane is $\left(g=10 \mathrm{~m} / \mathrm{s}^2\right)$

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Options:

A. $\sqrt{70} \mathrm{~m} / \mathrm{s}$

B. $\sqrt{\frac{140}{3}} \mathrm{~m} / \mathrm{s}$

C. $\sqrt{\frac{280}{3}} \mathrm{~m} / \mathrm{s}$

D. $\sqrt{100} \mathrm{~m} / \mathrm{s}$

Answer: D

Solution:

Given, height of inclined plane, $h=7 \mathrm{~m}$

$g=10 \mathrm{~m} / \mathrm{s}^2$

By conservation of energy,

Potential energy lost by the solid sphere in rolling down the inclined plane $=$ Kinetic energy gained by the sphere.

$$
\begin{aligned}
m g h & =\frac{1}{2} m v^2+\frac{1}{2} l \omega^2 \\
& =\frac{1}{2} m v^2+\frac{1}{2} \cdot \frac{2}{5} m R^2 \cdot\left(\frac{v}{R}\right)^2 \\
& =\frac{1}{2} m v^2+\frac{1}{5} m v^2 \\
m g h & =\frac{7}{10} m v^2 \\
\therefore \quad v^2 & =\frac{10}{7} g h \\
\Rightarrow \quad v & =\sqrt{\frac{10}{7} g h}=\sqrt{\frac{10}{7} \times 10 \times 7}=\sqrt{100} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Question132

Three identical rods each of mass ' M ' and length ' L ' are joined to form a symbol ' H. The moment of inertia of the system about one of the sides of ' H ' is

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Options:

A. $\frac{2ML^2}{3}$

B. $\frac{ML^2}{2}$

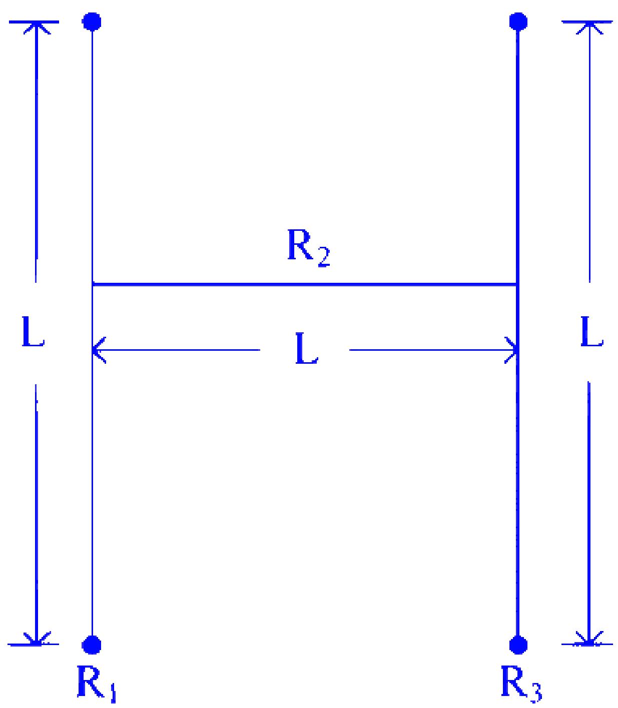
C. $\frac{ML^2}{6}$

D. $\frac{4ML^2}{3}$

Answer: D

Solution:

The given situation can be shown as



Let us take the moment of inertia of the system about rod R_1 then total moment of inertia is

$I_T = I_1 + I_2 + I_3 \quad .... \text{(i)}$

For rod $R_1, I_1 = 0$

For rod R_2, using perpendicular axis theorem,

$I_2 = \frac{ML^2}{3}$

For rod R_3, using parallel axis theorem,

$I_3 = I_{CM} + I_{(atL)} = 0 + ML^2 = ML^2$

Now, putting the values of I_1, I_2 and I_3 in Eq. (i), we get

$I_T = 0 + \frac{ML^2}{3} + ML^2 \Rightarrow I_T = \frac{4ML^2}{3}$

Question133

Three point masses each of mass ' m ' are kept at the corners of an equilateral triangle of side. The system rotates about the center of the triangle without any change in the separation of masses during rotation. The period of rotation is directly proportional to $\left(\cos 30^{\circ} = \sin 60^{\circ} = \frac{\sqrt{3}}{2}\right)$

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Options:

A. $\sqrt{L}$

B. $L^{3/2}$

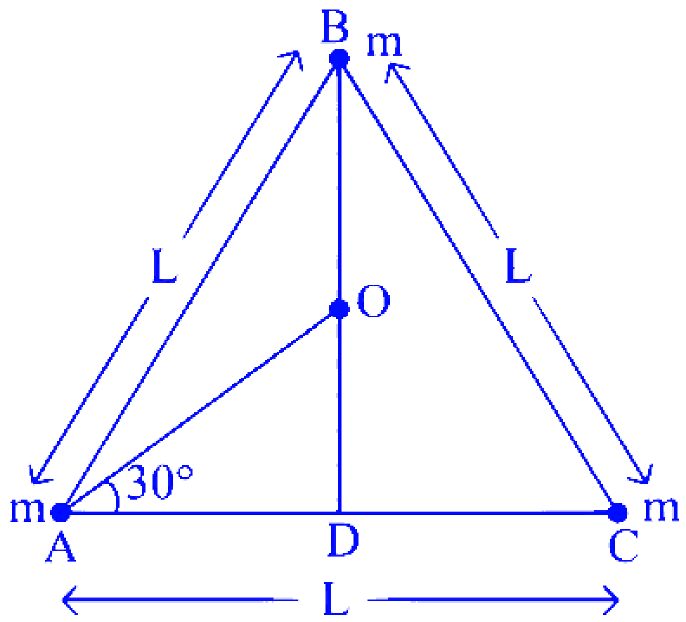
C. L

D. L^{-2}

Answer: A

Solution:

The situation can be shown below,



where, O is the centroid of equivalent triangle.

The length AO can be found by

$$AO = \sqrt{OD^2 + AD^2}$$
$$= \sqrt{\left(\frac{1}{3}BD\right)^2 + AD^2} = \sqrt{\frac{1}{9} \times \left(\frac{\sqrt{3}}{2}L\right)^2 + \frac{L^2}{4}}$$
$$= \sqrt{\frac{1}{12}L^2 + \frac{L^2}{4}} = \frac{L}{\sqrt{3}}$$

The moment of inertia about O is given by

$$I = MR^2$$
$$= m \times \left(\frac{L}{\sqrt{3}}\right)^2 = \frac{mL^2}{3} \quad .... \text{(i)}$$

According to law of conservation of angular momentum,

$$I\omega = \text{constant}$$
$$I \times \frac{2\pi}{T} = \text{constant}$$
$$\Rightarrow \quad T \propto I$$

From Eq. (i), we get

$$T \propto L^2$$

Question134

When a 12000 joule of work is done on a flywheel, its frequency of rotation increases from 10 Hz to 20 Hz . The moment of inertia of flywheel about its axis of rotation is $\left(\pi^2=10\right)$

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Options:

A. $1 \mathrm{~kgm}^2$

B. $2 \mathrm{~kgm}^2$

C. $1.688 \mathrm{~kgm}^2$

D. $1.5 \mathrm{~kgm}^2$

Answer: B

Solution:

Given, work done, $W=12000 \mathrm{~J}$,

Initial frequency, $f_1=10 \mathrm{~Hz}$

and final frequency, $f_2=20 \mathrm{~Hz}$

Angular velocity for rotational motion is given by

$$\begin{aligned} & \omega=2 \pi f \\ & \therefore \quad \omega_1=2 \pi t_1=2 \pi \times 10=20 \pi \mathrm{rad} / \mathrm{s} \\ & \text { and } \omega_2=2 \pi f_2=2 \pi \times 20=40 \pi \mathrm{rad} / \mathrm{s} \end{aligned}$$

According to work-energy theorem,

work done in rotation = change in rotational kinetic energy

$$\begin{aligned} \Rightarrow \quad W & =\frac{1}{2} l \omega_2^2-\frac{1}{2} l \omega_1^2 \quad\left[\because K E_{\text {rotational }} \quad=\frac{1}{2} / \omega^2\right] \\ & =\frac{1}{2} l\left(\omega_2^2-\omega_1^2\right) \quad \ldots .(\mathrm{i}) \end{aligned}$$

where, $I=$ moment of inertia of the flywheel.

Substituting given values in Eq. (i), we get

$$12000 = \frac{1}{2} l \left(1600\pi^2 - 400\pi^2\right)$$

$$\Rightarrow \quad = \frac{1}{2} l (1200 \times 10) \quad \left[\because \pi^2 = 10\right]$$

$$\Rightarrow \quad I = \frac{12000 \times 2}{12000} = 2 \text{ kg m}^2$$

Question135

A rigid body is rotating with angular velocity ' ω ' about an axis of rotation. Let v ' be the linear velocity of particle which is at perpendicular distance ' r ' from the axis of rotation. Then the relation $v = r\omega$ ' implies that

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Options:

A. ω does not depend on r

B. $\omega \propto \frac{1}{r}$

C. $\omega \propto r$

D. $\omega = 0$

Answer: A

Solution:

Using the given relation,

$v = r\omega$

If the perpendicular distance r from the axis of rotation is increased or decreased, then the value of linear velocity v accordingly increases or decreases. Thus, the value of angular velocity ω does not depend on r.

Question136

If radius of the solid sphere is doubled by keeping its mass constant, the ratio of their moment of inertia about any of its diameter is

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Options:

A. 1 : 8

B. 2 : 5

C. 2 : 3

D. 1 : 4

Answer: D

Solution:

Key Idea Moment of inertia of a solid sphere about its diameter is given as, $I=\frac{2}{5} M R^2$, where M and R are the mass and radius of the sphere, respectively.

Moment of inertia of a solid sphere about its diameter is $I=\frac{2}{5} M R^2$ .... (i)

As the radius of a solid sphere is doubled then the new moment of inertia of the sphere will be

$I'=\frac{2}{5} M(2 R)^2=\frac{8}{5} M R^2$ .... (ii)

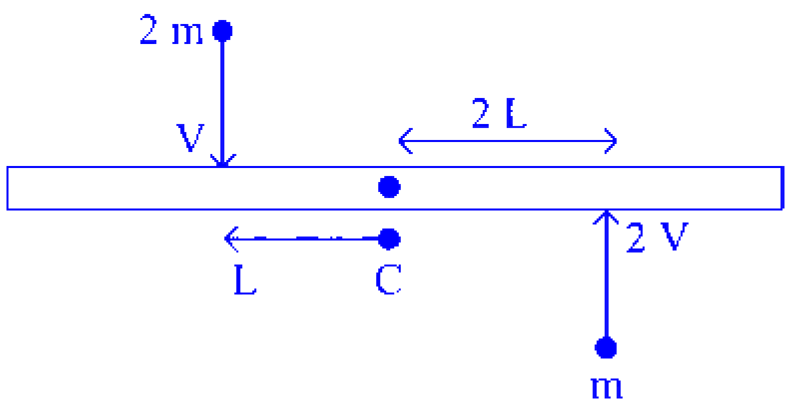
From Eqs. (i) and (ii), we get

$\frac{I}{I'}=\frac{\frac{2}{5} M R^2}{\frac{8}{5} M R^2}=\frac{1}{4}=1: 4$

The ratio of their moment of inertia about any of its diameter is 1 : 4.

Question137

A uniform rod of length ' 6 L ' and mass ' 8 m ' is pivoted at its centre ' C '. Two masses ' m ' and ' $2 m'$ with speed $2 v, v$ as shown strikes the rod and stick to the rod. Initially the rod is at rest. Due to impact, if it rotates with angular velocity ' ω ' then ' ω ' will be



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Options:

A. $\frac{V}{5L}$

B. zero

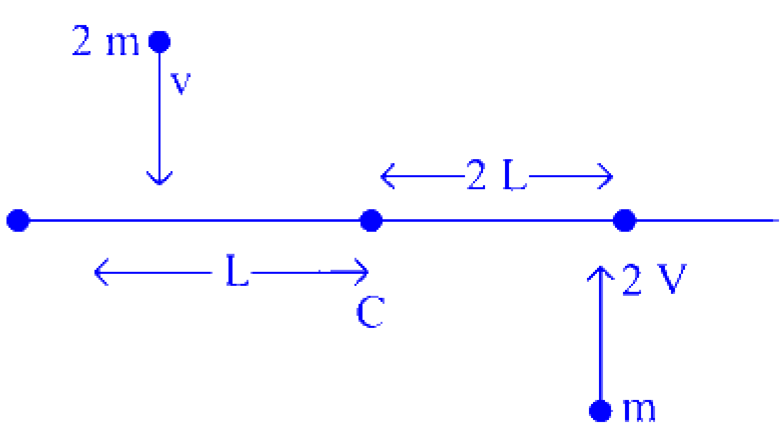
C. $\frac{8v}{6L}$

D. $\frac{11v}{3L}$

Answer: A

Solution:

A rod of length 6L and mass 8 m is given in figure,



When two masses strike the rod, then angular momentum imparted to rod,

$$\begin{aligned} L_1 + L_2 &= 2mv(L) + m(2v)(2L) \\ &= 6mvL \end{aligned}$$

Now, after striking of masses to rod the angular momentum of complete rod about the centre C,

$L_{\text{od}} \quad = 10 \quad$ ... (i)

where, $\omega =$ angular velocity of the rod

and $\quad I =$ moment of inertia of the rod.

The moment of inertia of rod

$I = \frac{MI^2}{12} = \frac{8m(6L)^2}{12} = 24mL^2$

$\therefore M = 8$ m and $I = 6L$ (given)

Now, moment of inertia of two masses after the striking to I rod

$I_1 = 2m(L)^2 = 2mL^2$

and $I_2 = m(2L)^2 = 4mL^2$

$\therefore$ The net moment of the inertia about the centre of rod,

$$I = 24mL^2 + 2mL^2 + 4mL^2$$
$$\Rightarrow \quad I = 30 \text{ mL}^2$$

By putting this value in Eq. (i), we get

$L_{\text{rod}} \quad = 30ML^2\omega$

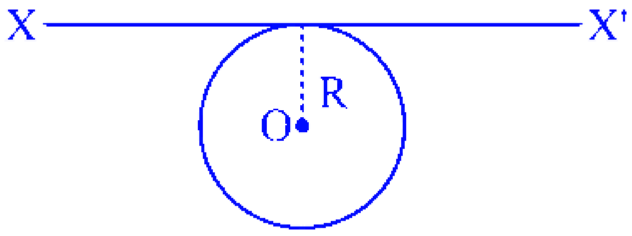
From the law of conservation of angular momentum,

$$L_1 + L_2 = L_{\text{rod}}$$
$$\Rightarrow \quad 6mvL = 30mL^2\omega$$
$$\Rightarrow \quad \omega = \frac{v}{5L}$$

Hence, the anguler velocity of the rod is $\frac{v}{5L}$.

Question138

A thin metal wire of length 'L' and uniform linear mass density 'ρ' is bent into a circular coil with 'O' as centre. the moment of inertia of a coil about the axis XX' is



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Options:

A. $\frac{3\rho L^3}{8\pi^2}$

B. $\frac{\rho L^3}{4\pi^2}$

C. $\frac{3\rho L^2}{4\pi^2}$

D. $\frac{\rho L^3}{8\pi^2}$

Answer: A

Solution:

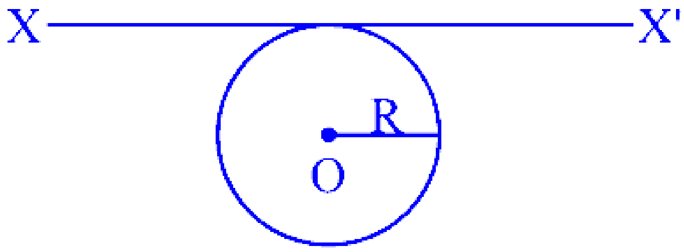
Key Idea Moment of inertia of a thin circular coil about its diameter,

$I=\frac{MR^2}{2}$

Moment of inertia of a thin circular coil,

$I=\frac{MR^2}{2}$

Now, moment of inertia of a ring about axis XX' as in figure below,



$I_{Xx'}=\frac{MR^2}{2}+MR^2=\frac{3}{2}MR^2$ .... (i)

(Using theorem of parallel axis)

Given, $L=$ length of wire of ring

and $\rho=$ linear mass density

Then, mass of the ring $=$ linear density $\times$ length

$\Rightarrow \quad M=\rho L$ ... (ii)

and $\quad L=2\pi R$

$\Rightarrow \quad R=\frac{L}{2\pi}$ ... (iii)

Now, putting the value from Eqs. (ii) and (iii) in (i), we get

$I_{xx'}=\frac{3}{2}(\rho L)\frac{L^2}{4\pi^2}\Rightarrow\frac{3\rho L^3}{8\pi^2}$

Hence, option a is correct.

Question139

The dimensions of torque are same as that of

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Options:

A. moment of force

B. pressure.

C. acceleration

D. impulse

Answer: A

Solution:

Torque is expressed as,

$\tau=$ force $(F)\times$ perpendicular distance (r)

So,

$$\begin{aligned}[\tau]=[F]\times[r] & =\left[\mathrm{MLT}^{-2}\right][\mathrm{L}] \\ & =\left[\mathrm{ML}^2\,\mathrm{T}^{-2}\right]\end{aligned}$$

Now, the dimension of moment of force $=$ force $\times$ distance $=\left[\mathrm{MLT}^{-2}\right]\times\left[\mathrm{L}^{\mathbf{1}}\right]=\left[\mathrm{ML}^2\mathbf{T}^{-2}\right]$

(b) Dimension of pressure $=$ Force $\times[$ Area $]^{-2}$

$=\left[MLT^{-2}\right]\times\left[\mathrm{L}^{-2}\right]=\left[\mathrm{ML}^{-1}\,\mathrm{T}^{-2}\right]$

(c) Dimension of acceleration $=\left[M^0L^1T^{-2}\right]$

(d) Dimension of impulse $=[$ Force $]\times[$ time $]$

$$= [MLT^{-2}][T^{1}] = MLT^{-1}$$

As, the torque has dimension $[ML^2 T^{-2}]$, which is correctly matched by the dimensions of moment of force. So, option (a) is correct.
